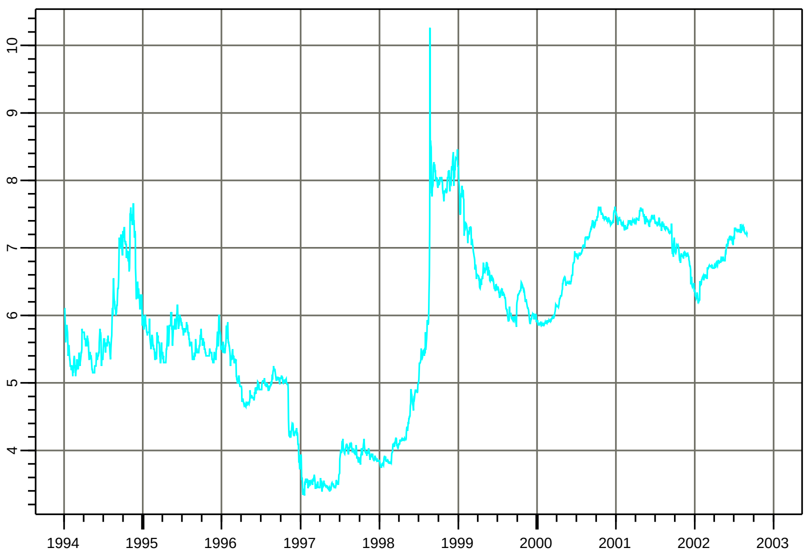
Modelling the stochastic behaviour of short-term interest rates: A survey



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Kjersti Aas

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Abstract:
Risk free interest rates play a fundamental role in finance. Theoretical models of interest rates are of interest both for the pricing of interest rate sensitive derivative contracts and for the measurement of interest rate risk arising from holding portfolios of these contracts. There is a vast literature focusing on modelling its dynamics. In this survey we describe some of the models.

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Norwegian Computing Center
Gaustadalléen 23, P.O. Box 114 Blindern, NO-0314 Oslo, Norway
Telephone: +47 2285 2500, telefax: +47 2269 7660, `http://www.nr.no`

Contents

1 Introduction **1**

2 Single-factor models **1**

3 GARCH-models **2**

4 Combined level-GARCH models **4**

5 Comparison of models **5**

6 Experiments **7**

References **9**

1 Introduction

Risk free interest rates play a fundamental role in finance. Theoretical models of interest rates are of interest both for the pricing of interest rate sensitive derivative contracts and for the measurement of interest rate risk arising from holding portfolios of these contracts. Consequently, an enormous amount of work has been directed towards modelling and estimation of the short-term interest rate dynamics in recent years. The dynamics of the short-term interest rate are rather complex, and there is today no consensus on how to model the rate and particularly its volatility. Examples of models that have been proposed in the literature are: single-factor diffusion, GARCH, regime-switching, and jump-diffusion models. This survey contains a non-comprehensive review of the models of the two first categories, as well as the group of models that combine these two types.

In Section 2 we review some of the single-factor models that have been proposed in the literature. These models parameterise the volatility only as a function of interest rate level, which is why they are often denoted level-models. These models fail to capture adequately the serial correlation in conditional variances. Time-varying conditional volatility patterns in finance are typically represented by the GARCH-type of models. However, unlike most financial time series, interest rates display conditional volatility patterns that are not only a function of past interest rate shocks, but are also considered as some function of the lagged level of the series itself. GARCH-models fail to capture this relationship, and as a result, fitting GARCH-models to interest rate series gives non-stationary models and explosive volatility patterns. In Section 3 we will discuss these issues further.

Lately, there have been attempts of combining the single factor and the GARCH models when modelling interest rates. In Section 4 we review some of these approaches, which have been denoted level-GARCH models (Andersen and Lund, 1997; Brenner et al., 1996), two-factor models (Bali, 2003; Longstaff and Schwartz, 1992) as well as mixed models (Rodrigues and Rubia, 2003). Most authors who have developed level-GARCH models compare their model to pure level- and pure GARCH-models, respectively. In Section 5 we review some of the results that have been reported. Finally, in Section 6, we report the results of fitting three of the models to the Norwegian 1-month and 3-month interest rates.

2 Single-factor models

Although they might be too simple to correctly model the extremely complex dynamics displayed by interest rates, single-factor models are widely used in practice because of their tractability and their ability to fit reasonably well the dynamics of the short term interest rates. The most general model in the one-factor class was proposed by Chan et al. (1992)

and is given by [1]

$$\begin{aligned} r_t &= \alpha_0 + (1+\alpha_1)\, r_{t-1} + \epsilon_t, \\ \mathrm{E}(\epsilon_t) &= 0 \\ \mathrm{Var}(\epsilon_t) &= \sigma^2\, r_{t-1}^{2\gamma}. \end{aligned} \tag{1}$$

This model provides a simple description of the stochastic nature of interest rates that is consistent with the empirical observation that interest rates tend to be mean-reverting. The parameter α_1 determines the speed of mean-reversion towards the stationary level. Situations where current interest rates are high, imply a negative drift until rates revert to the long-run value, and low current rates are associated with positive expected drift. It can also be noted that the variance of this process is proportional to the level of the interest rate; as the interest rate moves towards 0, the variance decreases. The parameter $\gamma \geq 0$ denotes the elasticity of the volatility against the level of the interest rate.

A large number of well-known models from the literature are particular cases of the model in Equation 1. If one sets $\gamma = 0$, one gets the Ornstein-Uhlenbeck process (Vasicek, 1977) (an autoregressive model of order 1 in the discrete case). Setting $\gamma = 1/2$ gives the CIR-model (Cox et al., 1985). Chan et al. (1992) have compared eight different models obtained by varying the the values of α_0, α_1, and γ. Their conclusion is that the most successful models in capturing the dynamics of the short-term interest rate are those that allow the volatility of the interest rate changes to be highly sensitive to the level of the interest rate. Moreover, they find that for one-month US Treasury bill yields, models with $\gamma \geq 1$ capture dynamics of the short-term interest rate better that models with $\gamma \leq 1$. For the one-month US Treasury bill rate, the best estimate of γ is found to be approximately 1.5 (the Generalized Method of Moments is used to estimate the model). According to Koedijk et al. (1997) this value of γ is so large that stationarity of the interest rate process is not guaranteed. Moreover, Brenner et al. (1996) claim that this very high value of γ is a result of model misspecification in terms of the conditional variance.

The main weakness of the single-factor model seems to be that it does not properly capture the serial correlation in the conditional variance of the interest rates. A different class of models to capture volatility dynamics, is the family of GARCH-models. Key ingredients in these models are volatility clustering and volatility persistence. In Section 3 we discuss the issue of fitting such models to short-term interest rates.

3 GARCH-models

Returns from financial market variables such as exchange rates, equity prices, and interest rates measured over short time intervals (i.e. intra-daily, daily, or weekly) are characterized by *volatility clustering*. Volatility clustering means that the volatility of the series varies over time. Small changes in the price tend to be followed by small changes, and large

[1] Interest rate models are most often presented on their continuous-time form. In this paper, we operate in the discrete time domain. It is important to acknowledge that the discretized version is only an approximation to the continuous-time specification.

changes by large ones. The success of the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) class of models (Bollerslev, 1986) at capturing volatility clustering for equity prices and interest rates is extensively documented in the litterature. Recent surveys are given in Ghysels et al. (1996) and Shepard (1996). Interest rates, however, display time-varying conditional volatility patterns that are not only a function of squared innovations that is the basic structure in the GARCH-models. As shown in Section 2, the volatility is also likely to be some function of the level of the interest rate. As a result, fitting the most common GARCH(1,1)-model to short-term interest rate series usually give non-stationary models. In this section we will discuss this issue a bit further.

A simple univariate GARCH(1,1)-model for interest rates can be written as follows:

$$\begin{aligned} r_t &= \alpha_0 + (\alpha_1 + 1)\, r_{t-1} + \epsilon_t, \\ \mathrm{E}(\epsilon_t) &= 0 \\ \mathrm{Var}(\epsilon_t) &= \sigma_t^2 \\ \sigma_t^2 &= \beta_0 + \beta_1\, \epsilon_{t-1}^2 + \beta_2\, \sigma_{t-1}^2, \end{aligned} \tag{2}$$

where $\epsilon_t, t = 1, \ldots,$ are serially independent. Equation 2 corresponds to Equation 1 in the particular case where $\gamma = 0$, and σ is dependent of t.

The parameters of the model for σ_t^2 satisfy $0 \leq \beta_1 \leq 1$, $0 \leq \beta_2 \leq 1$, and $\beta_1 + \beta_2 \leq 1$. The process is stationary if $\beta_1 + \beta_2 < 1$, and the stationary variance is given by $\beta_0/(1 - \beta_1 - \beta_2)$. The model is non-stationary if $\beta_1 + \beta_2 > 1$, i.e. volatility shocks persist forever. When fitting the GARCH(1,1) model to short-term interest rates, one often gets parameter estimates that correspond to non-stationary behaviour. For instance, Engle et al. (1990) estimates the sum to be 1.01 for U.S Treasury securities and Gray (1996) find the sum of the coefficients to be 1.03 for one-month T-bills.

Simulating from a non-stationary model may give explosive patterns of the interest rate. Several authors (Bali, 2003; Rodrigues and Rubia, 2003) claim that this behaviour is due to model misspecification, and that the specification in Equation 2 cannot be used for modelling interest rates, because it fails to capture the relationship between interest rate levels and volatility. In their opinion, both the level and the GARCH effects have to be incorporated into the conditional volatility process in order to determine the correct specification. During the last few years, models that encompass both the level effect of the single-factor models as well as the conditional heteroskedasticity effect of the GARCH-models have been presented(Bali, 2003; Brenner et al., 1996; Koedijk et al., 1997; Longstaff and Schwartz, 1992). In Section 4 we present some of these models.

It should be noted that there are GARCH-models that better fit short-term interest rates than the one above. Andersen and Lund (1997) report that the EGARCH class of models appears to perform satisfactorily for U.S. 3-month Treasury Bills with weekly resolution. In the EGARCH-model of Nelson (1990), the log-variance is modelled instead of the variance:

$$\log \sigma_t^2 = \beta_0 + \beta_1 \frac{|\epsilon_{t-1}| + \beta_3\, \epsilon_{t-1}}{\sigma_{t-1}} + \beta_2 \log \sigma_{t-1}^2. \tag{3}$$

An advantage with this model over the basic GARCH model is that the conditional variance, σ_t^2, is guaranteed to be positive regardless of the values of the coefficients. The stationary variance of this process is given by

$$\sigma^2 = \exp\left\{ \frac{\beta_0 + \beta_1 \sqrt{2/\pi}}{1-\beta_2} \right\}.$$

4 Combined level-GARCH models

Several papers have indicated that the one-factor models described in Section 2 tend to overemphasize the sensitivity of volatility to interest rate levels, and fail to capture adequately the serial correlation in conditional variances. On the other hand, as described in Section 3, the GARCH models that parameterise the volatility only as a function of unexpected interest rate shocks, fail to capture the relationship between interest rate levels and volatility. As a consequence, during the last few years, approaches that combine the two types of models have been presented. In this section we will describe some of these approaches (Bali, 2003; Brenner et al., 1996; Koedijk et al., 1997; Longstaff and Schwartz, 1992).

Longstaff and Schwartz (1992) present the following two-factor model

$$\begin{aligned} r_t &= \alpha_0 + (1+\alpha_1)\, r_{t-1} + \alpha_2\, \sigma_{t-1}^2 + \epsilon_t, \\ \mathrm{E}(\epsilon_t) &= 0 \\ \mathrm{Var}(\epsilon_t) &= \sigma_t^2 \\ \sigma_t^2 &= \beta_0 + \beta_1\, \epsilon_{t-1}^2 + \beta_2\, \sigma_{t-1}^2 + \beta_3\, r_{t-1}. \end{aligned} \tag{4}$$

Apart from the term $\beta_3\, r_{t-1}$, this is the GARCH-in-the-Mean model of Engle et al. (1987). Note that if $\beta_1 = 0$ and $\beta_2 = 0$, one gets:

$$r_t = (\alpha_0 + \alpha_2\, \beta_0) + (1 + \alpha_1 + \alpha_2\, \beta_3)\, r_{t-1} + \sqrt{\beta_0 + \beta_3}\, \sqrt{r_{t-1}}\epsilon_t,$$

which is the single-factor (CIR) model with $\sigma = \sqrt{\beta_0 + \beta_3}$ and $\gamma = 1/2$.

Brenner et al. (1996) proposed the following model:

$$\begin{aligned} r_t &= \alpha_0 + (1+\alpha_1)\, r_{t-1} + \alpha_2\, r_{t-1}^2 + \epsilon_t, \\ \mathrm{E}(\epsilon_t) &= 0 \\ \mathrm{Var}(\epsilon_t) &= \sigma_t^2\, r_{t-1}^{2\gamma} \\ \sigma_t^2 &= \beta_0 + \beta_1 \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}\, r_{t-2}^{\gamma}} \right)^2 + \beta_2\, \sigma_{t-1}^2 + \beta_3 \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}\, r_{t-2}^{\gamma}} \right)^2 I_{t-1}. \end{aligned} \tag{5}$$

where I_{t-1} is an indicator function that assumes value 1 when ϵ_{t-1} is negative and 0 when it is positive. That is, the model allows for an asymmetric volatility effect.

Ferreira (2000) suggest a very similar model, but where the log-variance is modelled instead of the variance:

$$\begin{aligned} r_t &= \alpha_0 + (1+\alpha_1)\, r_{t-1} + \epsilon_t, \\ \mathrm{E}(\epsilon_t) &= 0 \\ \mathrm{Var}(\epsilon_t) &= \sigma_t^2\, r_{t-1}^{2\gamma} \\ \log(\sigma_t^2) &= \beta_0 + \beta_1 \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}\, r_{t-2}^{\gamma}} \right)^2 + \beta_2\, \log(\sigma_{t-1}^2) + \beta_3 \left(\frac{\epsilon_{t-1}}{\sigma_{t-1}\, r_{t-2}^{\gamma}} \right)^2 I_{t-1}. \end{aligned} \tag{6}$$

In both models in Equations 5 and 6 the unscaled prediction errors $\left(\frac{\epsilon_{t-1}}{\sigma_{t-1}\, r_{t-2}^{\gamma}}\right)^2$ are used in the GARCH-equation. According to Rodrigues and Rubia (2003) this means that the volatility does not follow an ordinary GARCH(1,1)-model anymore. This makes it hard to establish the stationary conditions of this model (Koedijk et al., 1997). Other papers overcome this unappealing fact by slightly different models. First, Koedijk et al. (1997) propose the following model:

$$\begin{aligned}
r_t &= \alpha_0 + (1+\alpha_1)\, r_{t-1} + \alpha_2\, r_{t-1}^2 + \sigma_t\, r_{t-1}^{\gamma}\, \epsilon_t, \\
\mathrm{E}(\epsilon_t) &= 0 \\
\mathrm{Var}(\epsilon_t) &= \sigma_t^2\, r_{t-1}^{2\gamma} \\
\sigma_t^2 &= \beta_0 + \beta_1 \left(\frac{\epsilon_{t-1}}{r_{t-2}^{\gamma}}\right)^2 + \beta_2\, \sigma_{t-1}^2.
\end{aligned} \tag{7}$$

If $\alpha_2 = 0$, $\beta_1 = 0$ and $\beta_2 = 0$ one gets the single-factor model with $\sigma_t^2 = \beta_0$. Another special case is the GARCH-model, which is obtained if $\alpha_2 = 0$ and $\gamma = 0$. According to Rodrigues and Rubia (2003), in this model the GARCH parameters keep their standard interpretation, and stationarity is ensured if the usual parameter restrictions are met.

Finally, Bali (2003) gives a slightly more complicated model [2]:

$$\begin{aligned}
r_t &= (1+\alpha_1)\, r_{t-1} + \alpha_2\, r_{t-1}\, \log(r_{t-1}) + \frac{1}{2}\, \sigma_{t-1}\, r_{t-1} + \epsilon_t, \\
\mathrm{E}(\epsilon_t) &= 0 \\
\mathrm{Var}(\epsilon_t) &= \sigma_t^2\, r_{t-1}^{2\gamma} \\
\sigma_t^2 &= \beta_0 + \beta_1 \left(\frac{\epsilon_{t-1}}{r_{t-2}^{\gamma}}\right)^2 + \beta_2\, \sigma_{t-1}^2,
\end{aligned} \tag{8}$$

In the standard GARCH model in Section 3, $\beta_1 + \beta_2 > 1$ implies that current shocks affect volatility forecasts inifinitely far into the future. In the level-GARCH models, volatility persistence cannot be measured by $\beta_1 + \beta_2$, because the volatility is a function of both the stochastic volatility factor, σ_t and the interest rate levels. Hence, the persistence is a function of persistence in both σ_t and r_{t-1} (Bali, 2003).

5 Comparison of models

Most authors that have developed level-GARCH models compare their model to pure level- and pure GARCH-models, respectively. In this section we review some of the results that have been reported. Since the volatility does not follow an ordinary GARCH(1,1)-model in the models of Brenner et al. (1996) and Ferreira (2000) it is hard to establish the stationary conditions of these models. Hence, we concentrate on the methods of Longstaff and Schwartz (1992), Koedijk et al. (1997) and Bali (2003).

Koedijk et al. (1997) compare their model to a pure single factor model, to a pure GARCH-model and to the model proposed by Longstaff and Schwartz (1992), respectively,

[2]It is a two-factor extension of the single-factor model of Black et al. (1990)

for one month Treasury bill rates. They get the following parameter estimates for monthly data (the models are estimated by the method of quasi-maximum likelihood)[3]

Parameter	*Single-factor*	*GARCH*	*Koedijk*	*Longstaff*
$\alpha_0 \times 10$	0.01	-0.25	-0.30	-0.26
$\alpha_1 \times 10$	0.20	0.37	0.36	0.34
$\alpha_2 \times 10^2$	-0.58	-0.58	-0.44	-0.42
$\beta_0 \times 10^2$	0.13	0.77	0.02	-
β_1	-	0.26	0.18	0.25
β_2	-	0.75	0.74	0.70
β_3				0.31
γ	1.40	-	1.24	-
Log-L	-217	-214	-198	-208

While the pure GARCH-model is non-stationary in the variance with $\beta_1 + \beta_2 = 1.01$, the sum is only 0.92 for the model of Koedijk et al. (1997). Moreover, letting σ to be time-varying following its own dynamics, reduces the elasticity parameter γ from 1.4 to 1.24. The log-likelihood values indicate that both GARCH effects and the level effect are important determinants of interest rate volatility. The method of Koedijk et al. (1997) seems to be better than that of Longstaff and Schwartz (1992) in terms of the log-likelihood value[4].

The method of Bali (2003) has been tested on daily data on one-, three- and six-month Eurodollar interest rates. We only repeat the results for the three-month interest rates here. (All the models are fitted by the method of quasi-maximum likelihood).

Parameter	*Single-factor*	*GARCH*	*Bali*
α_1	-0.00229	-0.00210	-0.00195
α_2	-0.00094	-0.00063	-0.00059
$\beta_0 \times 100$	0.248	≈ 0	≈ 0
β_1	-	0.1813	0.2648
β_2	-	0.8486	0.8509
γ	1.35	-	0.1588
Log-L	44026	46009	46014

We see that in terms of the maximized log-likelihood values, the model (Bali, 2003) performs better than the two other models. However, compared to the study in (Koedijk et al., 1997) there are two striking differences. Firstly, the volatility component of the mixed model is not stationary ($\beta_2 + \beta_3 = 1.1$). Secondly, the relative differences of the γ-values between the single-factor and mixed models are much larger than those reported by Koedijk et al. (1997). This confirms what has previously been reported by others (e.g. Hong et al. (2004)), namely that the elasticity parameter is very sensitive to the choice of

[3]Koedijk et al. (1997) were not able to estimate the constant term, β_0, of the Longstaff-model when γ is a free parameter.

[4]Can compare models in terms of log-likelihood value since they are nested

Interest rate	Time period
NIBOR 1M	04.05.1993– 02.09.2002
NIBOR 3M	07.07.1992– 29.08.2002

Table 1: *Data. The series were given on daily resolution.*

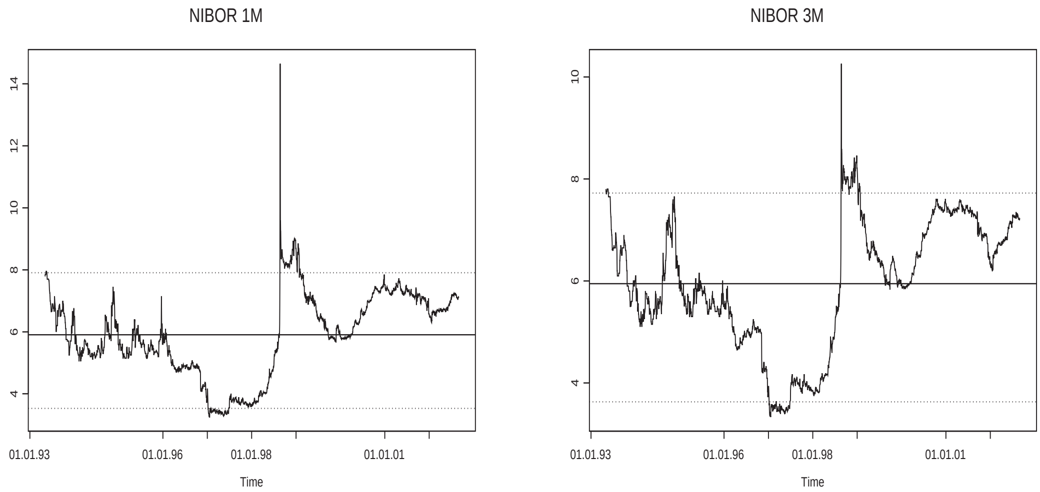

Figure 1: *Data.*

interest rate data, data frequency, sample periods, and the specifications of the volatility function.

6 Experiments

In this section we have compared some of the methods reviewed in this survey for the Norwegian interest rate series given in Table 1. Both series have a daily resolution. The series are shown in Figure 1. Table 2 shows the resulting parameter values when the original GARCH-model in Equation 2 is fitted to the interest rates [5]. All the parameter values are significant. For both series, $\beta_1 + \beta_2 > 1$, indicating non-stationary GARCH-models. Simulating from these models one is likely to get interest rates that “explode” in the long-run, which is an extremely erratic behaviour.

Next, we fitted a slightly modified version[6] of the Longstaff-Schwartz method to the logarithm of the interest rates. The estimated parameter values are shown in Table 3. Now the GARCH-model for NIBOR 3M is stationary. Moreover, the BIC criteria indicates that this model gives a better fit to the data than the original GARCH model. The GARCH-model for NIBOR 1M is however, still non-stationary.

[5] We have modelled the logarithm of the interest rates to avoid negative values.

[6] If we used the variance in the conditional mean equation, the parameter α_2 turned out to be non-significant. Hence, we used the standard deviation instead.

Finally, we fitted the EGARCH-model in Equation 3 to the two series. The results are given in Table 4. The leverage parameter β_3 was not significant for the NIBOR 1M series, but all other parameter values were significant. The BIC criteria indicates that this model gives a better fit to the data than the original GARCH-model, but not as good as the Longstaff-Schwartz model.

Interest rate	α_0	$\alpha_1 + 1$	β_0	β_1	β_2
NIBOR 1M	0.0037	0.994	2.80e-6	0.34	0.73
NIBOR 3M	0.0035	0.995	1.24e-6	0.27	0.78

Table 2: *Original GARCH(1,1)-model. All the parameter values are highly significant.*

Interest rate	α_0	$1 + \alpha_1$	α_2	β_0	β_1	β_2	β_3
NIBOR 1M	0.0057	0.992	-0.19	1.71e-6	0.28	0.73	-2.3e-5
NIBOR 3M	0.0053	0.993	-0.19	2.05e-5	0.26	0.70	-2.8e-5

Table 3: *Longstaff-Schwartz level-GARCH model. All the parameter values are significant.*

Interest rate	α_0	$1 + \alpha_1$	β_0	β_1	β_2	β_3
NIBOR 1M	0.0059	0.991	-0.746	0.45	0.95	-
NIBOR 3M	0.0057	0.992	-0.652	0.33	0.96	-0.05

Table 4: *EGARCH(1,1)-model. All parameter values are significant.*

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