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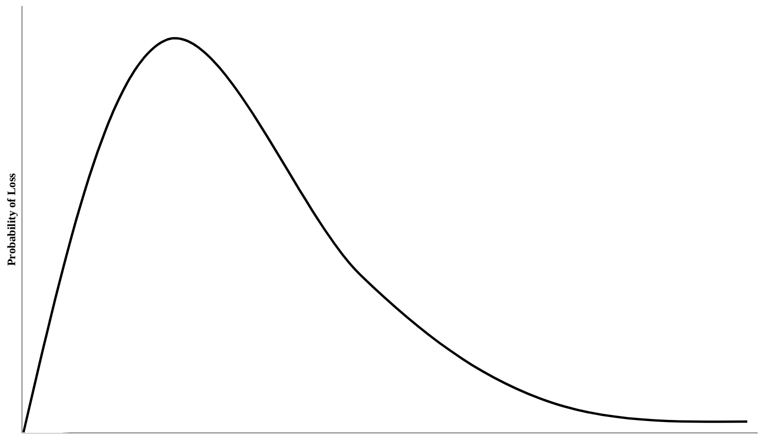
Note

The Basel II IRB approach for credit portfolios: A survey

Figure 1
Credit Loss Distribution



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Abstract

In this report we review the Basel II IRB approach, including the theory used to derive its model foundation, and the interpretation of its parameters. The IRB approach is a hybrid between a very simple statistical model of capital needs for credit risk and a negotiated settlement. It is characterised by its computational simplicity; there is an analytical formula for the calculation of capital, and the model is perfectly additive. However, this strength is also the cause of its weakness. The main assumptions have some negative consequences that are discussed in this report. Hence, the statistical model behind the Basel II IRB approach is not best practice today, and certainly will not become so in the future. Even the Basel Committee itself states that the IRB is work-in-progress, and will remain so long after the implementation of Basel II (Basel Committee on Banking Supervision, 2004b). We give some indications of how the IRB approach may be modified to deal with its weaknesses.

Keywords	Basel II advanced IRB approach, credit portfolio risk, default probability, loss given default, asset correlations
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Contents

1 Introduction

In June 2004, the Basel Committee issued a revised framework on international convergence of capital measurements and capital standards (Basel Committee on Banking Supervision, 2004b), which will serve as the basis for national rulemaking and implementation processes. The financial institutions may choose between two approaches to calculate the capital requirement for credit risk; the standardised approach (essentially a slightly modified version of the current accord) and the internal-ratings-based (IRB) approach. In the IRB approach, institutions are allowed to use their own measures for key drivers of credit risk as primary inputs to the capital calculation. Hence, this approach is regarded as a first step towards supervisory recognition of advanced credit risk models and economic capital calculations.

In the IRB approach, regulatory minimum capital for a credit risk portfolio is calculated in a bottom-up approach, by determining capital requirements on the asset level and adding them up. The capital requirements of assets are derived from risk weight formulas, which were developed considering a special credit portfolio model, the so-called Asymptotic Risk Factor (ASRF) model. Although there is no cited source or documentation behind this model, it is widely believed that the working paper version of Gordy (2003) was the precursor to the actual formulas. This model is characterised by its computational simplicity and the property that the risk weights of single credit assets depend only on the characteristics of these assets, but not on the composition of the portfolio (portfolio invariance).

In Chapter 2 we will give a description of the IRB approach, including the theory used to derive the model, and the interpretation of its parameters. The model framework for the advanced IRB approach is based on two main assumptions. These assumptions have some negative consequences that we will discuss in Chapter 3. We will also indicate how the IRB approach may be modified to deal with them.

2 The advanced IRB approach

In the advanced IRB approach, regulatory capital requirements for unexpected losses are derived from risk weight formulas, which are based on the so-called Asymptotic Risk Factor (ASRF) model. In this model, credit risk in a portfolio is divided into two categories, systematic and idiosyncratic risk. Systematic risk represents the effect of unexpected changes in macroeconomic and financial market conditions on the performance of borrowers. Idiosyncratic risk, on the other hand, represents the effects of risk connected to individual firms. The idea behind the ASRF model is that as the portfolio becomes more and more fine-grained, in the sense that the largest individual exposures account for a smaller and smaller share of total portfolio exposure, idiosyncratic risk is diversified away on the portfolio level. The great advantage of the ASRF model is that the capital charge for a lending exposure is based solely on loan-specific information. This allows one to calculate capital charges on a decentralised loan-by-loan basis first, and then aggregate these up to the portfolio-wide VaR afterwards. In Section 2.1 we review this model. In Section 2.2 we give the actual formula used to compute the economic capital requirement in the advanced IRB framework, and finally in Section 2.3 we describe the interpretation of the parameters in this framework.

2.1 The Asymptotic Risk Factor approach

The ASRF approach assumes that the bank credit portfolio consists of a large number of relatively small exposures. If this is the case, idiosyncratic risk associated with individual exposures tends to be cancelled out, and only systematic risks that affect many exposures have a material effect on portfolio losses. In the ASRF approach, all systematic risk, like industry or regional risk, is modelled with only one systematic risk factor. We first describe how default of a single firm is modelled in Section 2.1.1. Then, we outline how the single firm models can be used to derive a formula for the economic capital of the whole credit portfolio in Section 2.1.2. Finally, in Section 2.1.3 we give a slightly altered version of this formula, obtained when we group the clients into a set of relatively homogeneous subportfolios.

2.1.1 Default of one firm

The ASRF approach is derived from an adaptation of the single asset model of Merton (1974). In this approach, loans are modelled in a standard way as a claim on the value of a firm. The value of the firm's assets is measured by the price at which the total of the firm's liabilities can be purchased. Thus, the total value of the firm's assets is equal to the value of the stock plus the value of the debt. Loan default occurs if the market value of the firm's assets falls below the amount due to the loan. Thus the default distribution of a firm is a Bernoulli distribution, derived from the distribution of the value of the firm's asset returns.

We start by assuming that the normalised asset return R_i of firm i in the credit portfolio is driven by a single common factor Y and an idiosyncratic noise component ϵ_i:

$$R_i = \sqrt{\rho_i}\, Y + \sqrt{1-\rho_i}\, \epsilon_i, \tag{2.1}$$

where Y and ϵ_i are i.i.d. $N(0,1)$, meaning that R_i is considered to have a standardised Gaussian distribution. The component ϵ_i represents the risk specific to institution i and Y a common risk to all firms in the portfolio (representing the state of the macro economy). It should be noted that the interpretation of the R_i's as asset returns is merely intuitive, it is irrelevant to know the firms' true asset returns in this approach. It follows from Equation (2.1) that the assets of all firms are multivariate Gaussian distributed and the assets of two firms i and j are correlated with the linear correlation coefficient $E[R_i, R_j] = \sqrt{\rho_i}\sqrt{\rho_j}$ [1]. Moreover, the correlation between the asset return R_i and the common factor Y is $\sqrt{\rho_i}$. Hence, $\sqrt{\rho_i}$ is often interpreted as the sensitivity to systematic risk.

We define a binary random variable Z_i for each firm, which takes on value 1 (meaning that the ith obligor defaults) with probability p_i and value 0 with probability $1-p_i$. According to the theory of Merton (1974), we have

$$Z_{i,k} = 1 \text{ if } R_{i,k} \leq \Phi^{-1}(p_k) \quad \text{and} \quad Z_{i,k} = 0 \text{ if } R_{i,k} > \Phi^{-1}(p_k),$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard Gaussian distribution.

The parameter p_i is the unconditional default probability of obligor i. If the outcome of the systematic risk factor was known, we could calculate the conditional probability of default by

$$\begin{aligned} P(Z_i = 1|Y = y) &= P(R_i \leq \Phi^{-1}(p_i)|Y = y) \\ &= P(\sqrt{\rho_i}\, Y + \sqrt{1-\rho_i}\, \epsilon_i \leq \Phi^{-1}(p_i)|Y = y) \\ &= P(\epsilon_i < \frac{\Phi^{-1}(p_i) - \sqrt{\rho_i}\, Y}{\sqrt{1-\rho_i}}|Y = y) \\ &= \Phi\left(\frac{\Phi^{-1}(p_i) - \sqrt{\rho_i}\, y}{\sqrt{1-\rho_i}}\right). \end{aligned}$$

1. It should be noted that asset return correlations and default correlations are not the same. Typically, the default correlation is much smaller than the asset correlation (Frey et al., 2001b), see Appendix B for how they are related.

2.1.2 Portfolio loss

Vasicek (1977) showed that under certain conditions, Merton's single asset model can be extended to a model for the whole portfolio. The portfolio model used in the advanced IRB approach (Gordy, 2003; Pykhtin and Dev, 2002) is very similar to Vasicek's model. Inputs supplied by the bank include the exposure at default (EAD), the probability of default (PD), the loss given default (LGD) and the effective remaining maturity (M). Given these inputs, the IRB capital charge is computed by calculating capital charges on a decentralised loan-by-loan basis, and then aggregating these up to a portfolio-wide capital charge. In what follows, we describe the IRB approach in more detail.

Assume that we have a portfolio with N clients with different exposures E_i, asset correlations ρ_i, probabilities of default p_i and loss given defaults s_i. Define the exposure weight w_i of client i by $w_i = E_i / \sum_{i=1}^{N} E_i$, and let the portfolio loss per dollar of exposure be given by

$$L = \sum_{i=1}^{N} w_i \, s_i \, Z_i.$$

Provided that the clients depend on the same unique risk factor and no obligor accounts for a significant fraction of the portfolio, it can be shown (see Appendix A.1) that as the number of clients in the portfolio $N \to \infty$, the α-percentile of the resulting portfolio loss distribution approaches the asymptotic value

$$q_\alpha(L) = \sum_{i=1}^{N} w_i \, s_i \Phi \left(\frac{\Phi^{-1}(p_i) + \sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}} \right). \tag{2.2}$$

Note that what is actually computed here is *not* the total capital charge, but the capital charge per dollar of exposure.

Even though no bank can have an infinite number of loans, the asymptotic nature of the results does not diminish their practical usefulness. Gordy (2000) shows that the quantile is well-approximated by its asymptotic value for homogeneous portfolios of reasonable size, and according to Schönbucher (2002a) the approximation error becomes unacceptable only if very low asset return correlations ($\rho < 1\%$), very few obligors (less than 500), or extremely heterogeneous exposure sizes (for instance one dominating obligor) are considered.

The remarkable property of Equation (2.2), apart from its simplicity is the asymptotic capital additivity; the total capital for a large portfolio of loans is the weighted sum of the marginal capitals for individual loans. In other words, the capital required to add a loan to a large portfolio depends only on the properties of that loan/obligor and not on the portfolio it is added to. If this is the case, the credit model is said to be portfolio invariant. Note that portfolio-invariance depends strongly on the asymptotic assumption, and especially on the assumption of a single systematic risk-factor. Moreover, portfolios that are not asymptotically fine-grained, in the sense that any single obligor represents a negligible share of the portfolio's total exposure, contain undiversified idiosyncratic risk. In this case, the marginal contributions to the economic capital depend on the rest of the portfolio.

2.1.3 Homogeneous sectors

Grouping the clients into relatively homogeneous subportfolios might be convenient from an administrative point of view. In this section, we show that this also results in relatively simple capital formulas. Assume that the credit portfolio of a financial institution can be split into K homogeneous subportfolios. Each subportfolio k contains n_k identical clients with respect to the exposure E_k, the asset correlation ρ_k, the probability of default p_k and the expected loss given default rate s_k. The loss L_k of subportfolio k is given by

$$L_k = E_k\, s_k \sum_{i=1}^{n_k} Z_{i,k}.$$

We further assume that all clients in the entire credit portfolio depend on the same unique risk factor Y[2]. This means that the total loss of the portfolio is given by

$$L = \sum_{k=1}^{K} L_k = \sum_{k=1}^{K} E_k\, s_k \sum_{i=1}^{n_k} Z_{i,k}.$$

If we assume that the number of clients in each subportfolio $n_1, ..., n_K \to \infty$, it can be be shown (see Appendix A.2) that the α-percentile of the resulting asymptotic portfolio loss distribution is given by

$$q_\alpha(L) = \sum_{k=1}^{K} E_k\, s_k \Phi\left(\frac{\Phi^{-1}(p_k) + \sqrt{\rho_k}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_k}}\right). \tag{2.3}$$

Hence, the α-percentile of the loss distribution of the entire portfolio is just the sum of the α-percentiles of the loss distributions of the subportfolios, meaning that the capital can be calculated separately for all subportfolios and then aggregated by simple addition. Vasicek (2002) shows that if the portfolio contains a sufficiently large number of loans without being dominated by a few loans, this approximation holds even for portfolios with unequal exposures.

2.2 The IRB economic capital formula

While it is never possible to know in advance the losses a bank will suffer in a particular year, a bank can forecast the average level of credit losses it can reasonably

2. This implies that the correlation between the asset returns in two different subportfolios k and l is given by $\sqrt{\rho_k} \times \sqrt{\rho_l}$.

expect to experience. These losses are referred to as expected losses (EL). Losses above the expected levels are usually referred to as unexpected losses (UL). Institutions know that these losses will occur now and then, but they cannot know in advance the time of their arrival and their severity. Banks are in general expected to cover their EL on an ongoing basis, e.g. by provisions and write-offs, because it represents just another cost component of the lending business. According to this concept, capital is only needed for covering unexpected losses. Hence, in Basel II, the banks are only required to hold capital against UL, meaning that the needed capital per dollar of exposure is given by

$$C_\alpha(L) = q_\alpha(L) - E(L) = \sum_{i=1}^{N} w_i\, s_i \left[\Phi\left(\frac{\Phi^{-1}(p_i) + \sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}} \right) - p_i \right], \tag{2.4}$$

since the expected loss per dollar of exposure for the whole credit portfolio is given by

$$E(L) = \sum_{i=1}^{N} w_i\, s_i\, p_i.$$

In Basel II, the confidence level α is set to 99.9%, i.e. an institution is expected to suffer losses that exceed its economic capital once in a thousand years on average.

2.2.1 Maturity adjustments

Credit portfolios consist of instruments with different maturities. Empirical evidence indicates that long-term credits are riskier than short-term credits. For instance, downgrades from one rating category to a lower one, are more likely for long-term credits. Moreover, maturity effects are stronger for obligors with low probability of default. Consistent with these considerations, the Basel Committee has proposed a maturity adjustment m_i to be multiplied with each term of the economic capital computed from Equation (2.4). The maturity adjustment for client i is given by (Basel Committee on Banking Supervision, 2004a)

$$m_i = \frac{1 + (M - 2.5)\, b(p_i)}{1 - 1.5\, b(p_i)},$$

where

$$b(p_i) = (0.11852 - 0.05478 \times \log(p_i))^2,$$

and M is the maturity. In Basel II, M is set to 2.5 years for all rating grades. The maturity adjustment is only done for the corporate borrower portfolio.

2.3 Parameters

The parameters to be determined for each client in the IRB approach are

- p_i, the probability of default for client i.

- s_i, the loss given default for client i, which is the percentage of exposure the bank might loose in case client i defaults.

- $\sqrt{\rho_i}$, the correlation between the asset return for client i and the common factor.

- E_i, the exposure at default for client i, which is an estimate of the amount outstanding in case client i defaults.

The institutions may choose between two different versions of the IRB approach; foundation IRB and advanced IRB. For any of the versions, the institutions are allowed to determine borrowers' probabilities of default (PD) in their own way. However, the institutions using the advanced IRB approach are also permitted to rely on their own estimates of loss given default (LGD) and exposure at default (EAD). In what follows, we describe the interpretation of the parameters in the Basel IRB framework.

2.3.1 Probability of default

The PDs are supposed to be obtained from the internal rating system of the bank. According to the Basel Accord, they should be average quantities, reflecting expected default rates under normal business conditions.

Two kinds of models for determining probabilities of default are commonly addressed in the literature; accounting based models and market based models. Discriminant analysis and logistic regression models belong to the first class. The popular Z-score (Altman, 1968) is based on linear discriminant analysis, while Ohlsons O-Score (Ohlson, 1980) is based on generalised linear models (GLM) with the logit link function. Newer accounting based models are founded on neural networks (Wilson and Sharda, 1994) and Generalised Additive Models (GAM) (Berg, 2004).

The market models are based on the value of a firm, set by the market. One well-known example is the Moody's KMV model. Stock prices are commonly used as proxies for the value of the firm. Hence, this kind of models require that firms are registered on a stock exchange, a circumstance which is not fulfilled for most small and medium sized borrowers.

2.3.2 Loss given default

In the capital formula in Equation (2.3) it is assumed that the loss given default rate (which is equal to one minus the recovery rate) is known and non-stochastic (Gordy, 2003). During an economic downturn, losses on defaulted loans are likely to be higher than under normal business conditions, because for instance collateral values may decline. Average loss severity figures over long periods of time can understate LGD rates during an economic downturn, and may therefore need to be adjusted upward to appropriately reflect adverse economic conditions. Hence, one should choose conservative values in order not to underestimate the risk of the portfolio, and the Basel II capital formula requires that a “downturn” LGD is estimated for each client/risk segment. Due to the evolving nature of bank practices in the area of loss given default quantification, the Basel Committee has not proposed a specific rule for estimating the LGDs. Instead the banks are required to provide their own estimates.

Note that since one is supposed to use a “downturn” LGD, the EL in Equation (2.4) is probably overestimated, since the “downturn” LGD will generally be higher than the average one. Moreover, the capital requirement for defaulted assets will be 0. According to the Basel Committee, the latter is not desirable, since it does not cover systematic uncertainty in realised recovery rates for these exposures. Hence, the committee has stated that an “average” LGD should be used for computing the EL for defaulted assets. For performing loans however, the “downturn” LGD should be used both to compute the loss percentile and the expected loss.

2.3.3 Asset correlations

The asset correlations needed as input in the ASRF model, determine in short, how the asset values of the borrowers depend on each other. It should be noted that asset correlations and default correlations are not the same (see in Appendix B how they are related to each other). The input correlations also specify how the asset values of the borrowers depend on the general state of the economy, represented by the systematic risk factor, Y.

In the IRB approach, the asset correlations are not to be estimated by the banks. Instead they should be determined by formulas given by the Basel Committee. These formulas are based on two empirical observations (Lopez, 2004);

· asset correlations decrease with increasing probability of default

· asset correlations increase with firm size.

This means that the higher the probability of default, the higher the idiosyncratic risk components of an obligor. Moreover, conditional on a certain probability of default, assets of small and medium sized enterprises are less correlated. Hence, if two companies of different size have the same PD, it follows that the larger one is assumed to have a higher exposure to the systematic risk factor. In other words, larger firms are more closely related to the general conditions in the economy, while smaller firms are more likely to default for idiosyncratic reasons. The Basel Committee has provided different formulas for computing the asset correlations for different business segments. These are given in Appendix C.

2.3.4 Exposure at default

Under the advanced IRB approach, banks are allowed to use their own estimates of expected Exposure At Default (EAD) for each facility. Conceptually, EAD consists of two parts, the amount currently drawn, and an estimate of future draw downs of available, but untapped credit. Estimates of potential future draw downs (i.e. how the client may decide to draw unused commitments) are known as credit conversion factors (CCFs). Since the CCF is the only random or unknown proportion of EAD, estimating EAD amounts to estimating this CCF. The CCF is generally believed to depend on both the type of loan and the type of borrower. For example CCFs for credit cards are likely to be different from CCFs for corporate credit. At present, literature on these issues as well as data sources are scarce, but some suggestions as to which characteristics of a credit should be taken into account in EAD estimation is given Basel Committee on Banking Supervision (2005).

3 The assumptions behind the IRB economic capital formula

The IRB economic capital formula given in Equation (2.4) is characterised by its computational simplicity; there is an analytical formula for the calculation of capital, and the model is perfectly additive. However, these strengths are also the cause of its weaknesses. The main assumptions have some negative consequences. In this chapter we will discuss these consequences and indicate how the IRB approach may be modified to deal with them. First, Section 3.1 treats the two most important underlying assumptions, and then Section 3.2 describes other requirements of the IRB approach.

3.1 The two main assumptions

The development of the Basel II IRB approach was subject to some important restrictions in order to fit supervisory needs. There should be an analytical formula for the calculation of capital. Moreover, the model should be perfectly additive, or portfolio invariant, in the sense that the capital required for any given loan should depend only on the risk of that loan and not on the portfolio it is added to. These requirements impose strict assumptions on the diversification achieved within a bank portfolio:

- It must be assumed that the bank's credit portfolio is **infinitely fine-grained**, in the sense that any single obligor represents a negligible share of the portfolio's total exposure.

- It is assumed that a **single, common systematic risk factor** drives all dependence across credit losses in the portfolio, i.e. the bank must be well-diversified across all geographic and industrial sectors in a large economy.

Both of these assumptions are not always met by banks, especially by smaller institutions. If the first assumption is violated, IRB capital requirements may understate the true capital requirements, while the second assumption could result in a potential overestimation of portfolio risk.

3.1.1 Assumption 1: Fine-grained portfolio

The asymptotic capital formula given by Equation (2.4) is strictly valid only for a portfolio with infinitesimally small weights on its largest exposures. If a few large companies constitute a relatively large part of the credit portfolio, there will be a residual of undiversified idiosyncratic (unsystematic) risk in the portfolio. No real-world portfolios are in fact fine-grained portfolio, and therefore, one might question the relevance of the asymptotic formula. The asymptotic capital formula have been derived under the assumption that all the idiosyncratic risk is completely diversified away. Hence, it must underestimate the true capital, since any finite-size portfolio carries some undiversified idiosyncratic risk. Gordy (2003) shows that the remaining unsystematic risk is inversely proportional to the effective number of loans. This form of credit concentration risk is sometimes known as *granularity*, and may be addressed via granularity adjustment to the economic capital. The Basel Committee recognised this initially and introduced a granularity adjustment in Basel Committee on Banking Supervision (2001). Since then, at least four different approaches to granularity adjustment have been proposed in the literature (Emmer and Tasche, 2005; Gordy, 2004; Martin and Wilde, 2002; Pykhtin and Dev, 2002).

As shown in Section 2.2, the economic capital formula doesn't give extended regulatory capital requirements for risk concentrations. However, according to the Basel II Accord banks will have to demonstrate to the supervisors that they have established appropriate procedures to keep concentrations under control. Therefore, there is a need for quantitative tools with which to monitor risk concentrations, as well as guidelines for capital buffers against such concentrations. In Sections 3.1.1 and 3.1.2, we have a closer look at the two assumptions.

3.1.2 Assumption 2: Single systematic factor

The problem associated with assuming a single systematic risk factor is perhaps more difficult to address analytically than the granularity risk, but has greater consequences, especially for institutions with broad geographical and sectoral diversification. A single-factor model cannot reflect segmentation effects, say by industry branches. This failure to recognise the diversification effects of segmentation could result in an overestimation of portfolio risk. Diversification is one of the key tools for managing credit risk, and it is of vital importance that the credit portfolio framework used to calculate and allocate credit capital effectively models portfolio diversification effects.

In reality, the global business cycle is composed of many small economic changes, which might be linked to geography (e.g. political shifts) or to prices of production inputs (e.g., oil). If different industries and geographic regions may experience different cycles, they should be presented by distinct, though possibly correlated, systematic risk factors. Recently, several authors have proposed such multifactor models, see e.g. Céspedes (2002); Céspedes et al. (2005); Chabaane et al. (2004); Hamerle et al. (2003); Pykhtin (2004); Schönbucher (2002a); Tasche (2005).

Moreover, commercial credit risk models like Creditrisk+ and the Portfolio Manager software of Moody's KMV, typically use multiple factors for determining portfolio risk. Hence, the theoretical justification for the single factor model is weak and does not correspond to best practice in banking.

3.2 Other assumptions

In this section we describe other requirements on which the IRB approach is based. First, the IRB approach assumes that the LGD rate is deterministic and independent of the PD, while there is a reason to believe that the opposite is the case. This issue will be treated in Section 3.2.1. Second, the asset correlations are determined from the PDs, using a downward sloping relationship which has been rejected by many recent studies. We will come back to this in Section 3.2.2. Finally, the IRB approach models the asset returns with a multivariate normal distribution. However, many empirical investigations, see e.g. Frey et al. (2001a) reject the normal distribution because it is unable to model dependence between extreme events. In Section 3.2.3 we will have a closer look at this issue.

3.2.1 Loss given default

The IRB approach assumes that the LGDs are fixed deterministic quantities. The argument for this is that the uncertainty of the LGD does not contribute significantly to the risk of the client when compared with the PD volatility. In reality however, the LGD is not a fixed quantity and in the literature, it is quite common to assume that the it is a random value between 0 and 1, see e.g. Altman et al. (2003); Frye (2000); Hu and Perraudin (2002); Pykhtin (2003). The beta distribution is often considered appropriate for modelling the LGD, see e.g. Gupton and Stein (2002), because it is bounded between two points and can assume a wide range of shapes.

Although the Basel II Accord emphasises the need to consider correlation of PD and LGD, the capital formula in Equation (2.3) is linear in LGD and implicitly assumes independence. This assumption is made to simplify the IRB approach, and has neither theoretical nor empirical support. Recently, several authors have stated that there is reason to believe that there is a dependence between default events and losses given default. For example, a negative cyclical downturn is likely to produce an increase in the number of defaults, and at the same time a decrease in the number of recoveries. The dependence between default events and recovery rates may for instance be introduced through a factor that drives both default events and recovery rates (Frye, 2000; Pykhtin, 2003).

3.2.2 Asset correlations

As can be seen from Equation (C.1), Basel II assumes a downward sloping relationship between the asset correlation and the probability of default, i.e. the correlation between the assets of low-risk firms is assumed to be *higher* than the ones between high-risk firms. Several studies have produced results that contradict this relationship, see e.g. (Dietsch and Petey, 2004; Düllmann and Scheule, 2003; Hamerle et al., 2003; Rösch, 2002). If in reality the two effects do not offset each other, or worse if they reinforce each other, this could lead to a gross underestimation of the size of extreme losses. While the negative relationship between the asset correlation and the probability of default seems to be rejected by many recent studies, the decreasing relationship of the asset correlation with decreasing firm size is generally supported, see e.g. Dietsch and Petey (2004); Düllmann and Scheule (2003).

It should also be noted that the economic capital formula in Equation (2.4) is very sensitive to the asset correlation parameters. In particular, if the asset correlations are a few percentage points higher than what Basel assumes for a high-risk client (see Appendix B for how the asset correlation and probability of default are related to each other), the required capital may be significantly higher. Similarly, if it is lower that what Basel says for low-risk segments, their capital can be drastically reduced. Hence, the degree of dependence between firms, i.e. the size of correlations in this model, very strongly influences portfolio risk, especially for high confidence levels α (Perli and Nayda, 2004; Wehrspohn, 2002).

An alternative to the formulas specified by the Basel Committee, is to estimate the asset correlations from historical data. A direct estimation of these parameters, however, is not possible since the asset values are not observable. This problem is sometimes bypassed by approximating asset returns by equity returns. However, this solution requires that firms are registered on a stock exchange, a circumstance which is not fulfilled for most small and medium sized borrowers. An alternative approach is to use observed data on default rates to estimate the asset correlations. Appendix C of Case (2003) presents a brief summary of empirical methods to estimate asset correlations in practice, both from historical default rates and from historical economic-return-on-asset data.

3.2.3 Asset distributions

The Basel credit risk portfolio model makes use of a multidimensional normal distribution. Many empirical investigations, see e.g. Frey et al. (2001a), reject the normal distribution because it is unable to model dependence between extreme events. The assumption of a joint Gaussian distribution for the obligors' asset returns has another weakness; it implies that it is very difficult to fit both the tails and the main body of the portfolio credit loss distribution well (Schönbucher, 2002b). Hence, other multivariate distributions and copulas have been considered, e.g. the multivariate Student's t- and normal inverse Gaussian distribution (Wehrspohn, 2002), the Student's t- and the grouped t-copula (Daul et al., 2003), other elliptical copulas (Frey et al., 2003; Schmidt, 2003) and Archimedean copulas (Frey et al., 2003).

4 Summary

In this report we have reviewed the Basel II IRB approach, including the theory used to derive its model, and the interpretation of its parameters. The IRB approach is a hybrid between a very simple statistical model of capital needs for credit risk and a negotiated settlement. It is characterised by its computational simplicity; there is an analytical formula for the calculation of capital, and the model is perfectly additive. However, these strengths are, at the same time, the cause of its weaknesses. The main assumptions have some negative consequences that have been addressed in this report. Hence, the statistical model behind the Basel II IRB approach is not best practice today, and certainly will not become so in the future. Even the Basel Committee itself states that the IRB is work-in-progress, and will remain so long after the implementation of Basel II (Basel Committee on Banking Supervision, 2004b). We have given some indications of how the IRB approach may be modified to deal with its weaknesses.

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A Proofs of asymptotic capital rule

In Section 2.1, we presented two important results concerning the α-percentile of the asymptotic portfolio loss distribution. The first, given by Gordy (2003); Pykhtin and Dev (2002) and others, states that the asymptotic α-percentile of the loss per dollar exposure of a portfolio with N clients with different exposure weights w_i, asset correlations ρ_i , probabilities of default p_i and expected loss given default rates s_i, is given by

$$q_\alpha(L) = \sum_{i=1}^{N} w_i\, s_i\, \Phi\left(\frac{\Phi^{-1}(p_i) + \sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}}\right) \tag{A.1}$$

when $N \to \infty$, i.e. the number of clients in the portfolio becomes large, provided the clients depend on the same unique risk factor and no obligor accounts for a significant fraction of the portfolio. An outline of the proof of this result is given in Appendix A.1.

The other, given by Schönbucher (2002a); Wehrspohn (2002) and others, states that the asymptotic α-percentile of the loss of a homogeneous portfolio with a total exposure of E distributed on N identical clients with respect to asset correlations ρ, probabilities of default p, and expected loss given default rates s is given by

$$q_\alpha(L) = E\, s\, \Phi\left(\frac{\Phi^{-1}(p) + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right), \tag{A.2}$$

when $N \to \infty$, provided the clients depend on the same unique risk factor and no obligor accounts for a significant fraction of the portfolio. The proof of this result is given in Appendix A.2.

A.1 Asymptotic loss for heterogeneous portfolio

In this section, we outline the proof of the capital formula given in Equation (A.1). For the full proof, see Gordy (2003). Assume that we have a portfolio with N clients with different exposures E_i, asset correlations ρ_i, probabilities of default p_i and LGDs s_i. Assume further that they all depend on the same systematic risk factor Y. Let the portfolio loss be given by

$$L = \sum_{i=1}^{N} E_i\, s_i\, Z_i,$$

where $Z_i = 1$ if obligor i defaults. Further define $U_i = s_i\, Z_i$ and assume that the U_is are bounded on the interval $[-1, 1]$, and, conditionally on Y, are mutually independent. Define the portfolio loss ratio L_R as the ratio of the total losses to the total portfolio exposure, i.e.,

$$L_R = \frac{\sum_{i=1}^{N} E_i\, s_i\, Z_i}{\sum_{i=1}^{N} E_i}.$$

Assume now that the share of the largest exposure in the portfolio vanishes to zero as the number of exposures in the portfolio increases. It then might be shown that

$$\lim_{n\to\infty} L_R - \mathrm{E}[L_R|Y] = 0, \tag{A.3}$$

where the conditional expectation is given by

$$\mathrm{E}[L_R|Y] = \frac{\sum_{i=1}^{N} E_i\, \mathrm{E}[U_i|Y]}{\sum_{i=1}^{N} E_i}. \tag{A.4}$$

Hence, in the limit, we need only know the distribution of $\mathrm{E}[L_R|Y]$ to answer questions about the distribution of L_R. The proof is given in Appendix A of Gordy (2003). It can also be shown that under the same assumptions

$$\lim_{n\to\infty} q_\alpha(L_R) - q_\alpha\left(\mathrm{E}[L_R|Y]\right) = 0. \tag{A.5}$$

Strictly speaking, this result does not always hold, but it is approximately true for all practical purposes. The proof is given in Appendix B of Gordy (2003). Equation (A.5) allows substituting the quantiles of $\mathrm{E}[L_R|Y]$ (which may be relatively easily to calculate) for the corresponding quantiles of the loss rato L_R (which are hard to calculate) as the portfolio becomes large. It should be emphasized that Equation (A.5) is obtained with minimal restrictions on the make-up of the portfolio. The assets may have quite varied PD, LGD and exposure sizes. Moreover, there is no restriction on the relationship between E_i and the distribution of U_i. Hence if, for instance, high-quality loans tend also to be the largest loans, the result is still valid.

The final result is that

$$q_\alpha\left(\mathrm{E}[L_R|Y]\right) = \mathrm{E}[L_R|q_\alpha(Y)]. \tag{A.6}$$

To obtain this result, one must make some assumptions concerning the behaviour of $\mathrm{E}[L_R|Y]$, to make sure that there is no complex dependence between the tail quantiles of the loss distribution and the way the conditional expected loss for each borrower varies with Y. The conditional expectation must for instance be non-decreasing in Y, see Gordy (2003) for the proof. The result in Equation (A.6) is important, because $q_\alpha\left(\mathrm{E}[L_R|Y]\right)$ in general may be quite complicated to compute, while Equation (A.4) states that $\mathrm{E}[L_R|q_\alpha(Y)]$ is simply the exposure-weighed average of the individual assets' conditional expected losses.

From Equations (A.4),(A.5) and (A.6) we have that

$$\lim_{n\to\infty} q_\alpha(L_R) = q_\alpha\left(\mathrm{E}[L_R|Y]\right) = \mathrm{E}[L_R|q_\alpha(Y)] = \frac{\sum_{i=1}^N E_i\,\mathrm{E}\left[U_i|Y=\Phi^{-1}(\alpha)\right]}{\sum_{i=1}^N E_i}. \qquad (A.7)$$

Further, since the LGDs are assumed to be known and deterministic (see Section 2.3.2)

$$\mathrm{E}\left[U_i|Y=\Phi^{-1}(\alpha)\right] = s_i\,\mathrm{E}\left[Z_i|Y=\Phi^{-1}(\alpha)\right] = s_i\,P\left(Z_i=1|Y=\Phi^{-1}(\alpha)\right),$$

and from Section 2.1.1 we have

$$P\left(Z_i=1|Y=\Phi^{-1}(\alpha)\right) = s_i\,\Phi\left(\frac{\Phi^{-1}(p_i)-\sqrt{\rho_i}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}}\right).$$

Inserting the expression for $\mathrm{E}\left[U_i|Y=\Phi^{-1}(\alpha)\right]$ into Equation (A.7) and the definition of w_i from Section 2.1.2, we get

$$\lim_{n\to\infty} q_\alpha(L_R) = \sum_{i=1}^N w_i\,s_i\,\Phi\left(\frac{\Phi^{-1}(p_i)-\sqrt{\rho_i}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}}\right),$$

which is equal to the formula in Equation (A.1).

A.2 Asymptotic loss for homogeneous portfolio

In what follows, we assume that we have a homogeneous portfolio containing N identical clients with respect to asset correlations ρ, probabilities of default p, and LGDs s. Moreover, the clients are assumed to depend on the same unique risk factor. The aim of this section is give the proof the capital formula in Equation (A.2). The proof is taken from Schönbucher (2002a).

The portfolio loss for a certain year is

$$L = E\,s\,X,$$

i.e. the loss fraction X multiplied with the total exposure of the portfolio, E, and the loss given default rate, s. Since the two latter are assumed to be deterministic quantities, it is sufficient to know the distribution of the loss fraction.

First, we are interested in determining the probability that exactly n out of the N clients will default. Define

$$p(y) = P(Z_i=1|Y=y) = \Phi\left(\frac{\Phi^{-1}(p)-\sqrt{\rho}y}{\sqrt{1-\rho}}\right), \qquad (A.8)$$

which is the probability of firm i defaulting, given that the systematic factor Y takes the value y. Then conditional on $Y=y$, the probability of having n defaults in the portfolio is binomially distributed:

$$P\left(\sum_{i=1}^N Z_i = n|Y=y\right) = \begin{pmatrix} N \\ n \end{pmatrix} (p(y))^n\,(1-p(y))^{N-n}. \qquad (A.9)$$

Using the law of iterated expectations, the probability of n defaults is the expected value of the conditional probability of n defaults, i.e.

$$
\begin{aligned}
P\left(\sum_{i=1}^{N} Z_i = n\right) &= E\left[P\left(\sum_{i=1}^{N} Z_i = n|Y=y\right)\right] && \text{(A.10)}\\
&= \int_{-\infty}^{\infty} P\left(\sum_{i=1}^{N} Z_i = n|Y=y\right)\phi(y)\,dy && \text{(A.11)}\\
&= \int_{-\infty}^{\infty} (p(y))^n\,(1-p(y))^{N-n}\,\phi(y)\,dy. && \text{(A.12)}
\end{aligned}
$$

With the probability of n defaults given as shown above, the distribution of the number of defaults is

$$
P\left(\sum_{i=1}^{N} Z_i \le m\right) = \sum_{n=0}^{m}\int_{-\infty}^{\infty} (p(y))^n\,(1-p(y))^{N-n}\,\phi(y)\,dy. \qquad \text{(A.13)}
$$

If N is very large, and the portfolio is infinitely granular (i.e. each exposure is set to be equal to $1/N$, with large N, a simplification of the model is possible (Schönbucher, 2002a). Conditional on a realisation y of the common factor Y, obligor defaults are independent of each other. Therefore, in a very large portfolio, the law of large numbers ensures that the fraction of clients defaulting in the portfolio given $Y=y$ is equal to the individual conditional default probability in Equation (A.8), i.e.

$$
P\left(\frac{1}{N}\sum_{i=1}^{N} Z_i = p(y)|Y=y\right) = 1.
$$

Hence, if we know Y, then we can predict the fraction of clients that will default with certainty.

We want to determine the distribution function of the loss fraction $X=\frac{1}{N}\sum_{i=1}^{N} Z_i$:

$$
\begin{aligned}
P\left(X \le x\right) &= E\left[P\left(X \le x|Y=y\right)\right]\\
&= \int_{-\infty}^{\infty} P\left(X \le x|Y=y\right)\phi(y)\,dy\\
&= \int_{-\infty}^{\infty} P\left(p(Y) \le x|Y=y\right)\phi(y)\,dy\\
&= \int_{-\infty}^{\infty} I\left(p(y) \le x\right)\phi(y)\,dy\\
&= \int_{-y^*(x)}^{\infty} \phi(y)\,dy\\
&= \Phi(y^*).
\end{aligned}
$$

Here, $y^*(x)$ must be chosen such that $p(-y^*(x)) = x$ [1], i.e.

$$
y^*(x) = \frac{\sqrt{1-\rho}\,\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}.
$$

1. We also have that $p(y)$ from Equation (A.8) decreases in y, such that $p(y) \le x$ for $y > -y^*(x)$.

Hence,

$$P\left(X \leq x\right) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}\right).$$

If the α-percentile of the loss fraction distribution is denoted x_α, we have that

$$\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(x_\alpha) - \Phi^{-1}(p)}{\sqrt{\rho}}\right) = \alpha,$$

meaning that

$$x_\alpha = \Phi\left(\frac{\Phi^{-1}(p) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right).$$

Hence, the α-percentile of the loss distribution is obtained as

$$q_\alpha(L) = E\,s\,\Phi\left(\frac{\Phi^{-1}(p) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right), \tag{A.14}$$

which is equal to the formula in Equation (A.2).

B Portfolio variance and default correlation

In general, the portfolio variance is the sum of the variances and covariances of the portfolio components. Assuming that the PDs, exposures, and LGDs are independent, and that the two latter are non-stochastic, the portfolio variance is given by

$$\begin{aligned}
\sigma^2 &= \sum_{i=1}^{N}\sum_{j=1}^{N} \kappa_{ij}\sigma_i\sigma_j \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} \kappa_{ij}\sqrt{w_i\, s_i\, p_i\,(1-p_i)}\sqrt{w_j\, s_j\, p_j\,(1-p_j)}
\end{aligned}$$

where N is the number of clients in the portfolio, and κ_{ij} is the default correlation between client i and j. In the Basel II model, the default correlation κ_{ij} can be computed from the asset correlations ρ_i and ρ_j using

$$\kappa_{ij} = \frac{\Phi_2\left(\Phi^{-1}(p_i), \Phi^{-1}(p_j), \sqrt{\rho_i}\sqrt{\rho_j}\right) - p_i\, p_j}{\sqrt{p_i\,(1-p_i)\, p_j\,(1-p_j)}}. \tag{B.1}$$

Here $\Phi_2(\cdot)$ is the bivariate normal cumulative distribution function.

C Asset correlations in the IRB approach

The Basel Committee has provided different formulae for the asset correlations for different business segments. We give the formulae for bank, sovereign and corporate borrowers in Section C.1, and for retail portfolios in Section C.2.

C.1 Corporate, sovereign, interbank

The asset correlation function used for bank and sovereign exposures is given by

$$\rho_i = 0.12 \times \frac{1-e^{-50p_i}}{1-e^{-50}} + 0.24 \times \left(1 - \frac{1-e^{-50p_i}}{1-e^{-50}}\right). \tag{C.1}$$

According to the Basel Committee (Basel Committee on Banking Supervision, 2004a), the proposed shape of the above exponentially decreasing functions are in line with the findings of empirical studies. For corporate borrowers, the correlations ρ are first computed by Equation (C.1) and then modified as follows

$$\begin{aligned} \rho_i - 0.04, &\qquad S_i \leq 5 \text{ EUR} \\ \rho_i - 0.04 \times (1 - (S_i - 5)/45), &\quad 5 \text{ EUR} < S \leq 50 \text{ EUR} \\ \rho &\qquad S_i > 50 \text{ EUR}. \end{aligned}$$

Here, the S_i is annual sales for firm i. Hence, for small firms the asset correlations are lowered.

C.2 Retail

The Basel Committee has also provided specific mappings between probability of default p and asset correlation ρ for the retail portfolios. The correlation formulae, which have an empirical basis (Basel Committee on Banking Supervision, 2004a), are as follows:

· Residental mortgages

$$\rho_i = 0.15,$$

· Qualifying revolving retail exposures

$$\rho_i = 0.04,$$

- Other retail exposures

$$\rho_i = 0.03 \times \frac{1 - e^{-35p}}{1 - e^{-35}} + 0.16 \times \left(1 - \frac{1 - e^{-35p}}{1 - e^{-35}}\right).$$