TIME SERIES ANALYSIS OF UNEQUALLY SPACED OBSERVATIONS - WITH APPLICATIONS TO COPPER CONTAMINATION OF THE RIVER GAULA IN CENTRAL NORWAY

MAGNE ALDRIN, EIVIND DAMSLETH and HANS VIGGO SÆBØ
Norwegian Computing Center, P.O. Box 114, Blindern, N-0314 Oslo 3, Norway

(Received March 1989)

Abstract. The upper parts of the river Gaula in Central Norway are heavily contaminated by toxic metals – particularly copper (Cu).

A monitoring program for the river was established in early 1986, and the concentration of Cu, among other variables, has been measured.

There is a fairly strong temporal component in the Cu measurements, which calls for some sort of time series model. The irregular pattern of the observation times, however, makes the usual models infeasible, as they assume equi-spaced observations.

In the paper we present a simple DLM (Dynamic Linear Model) which gives a satisfactory description of the Cu concentration series. The model is fitted to the data using a Kalman filter technique which handles the irregularly spaced observations without problems.

We have utilized the model to interpolate non-observed values, estimate the net loading and to simulate alternative patterns for the Cu series, observed values and the runoff, to obtain estimates of the extreme values and the probability that the concentration has exceeded a certain limit.

1. Introduction

The problem of estimating various characteristics (mean, median, etc.) of the concentration and/or loadings of various contaminants, given limited amounts of data, has been given considerable attention both from water chemists ans statisticians. The purpose of the present paper is to give a brief review of some more or less well known techniques, their advantages and their shortcomings, and to illustrate the techniques with an application from river Gaula in Central Norway.

The outline of the paper is as follows: In Section 2 we give a brief presentation of river Gaula and its surroundings. Section 3 describes the monitoring program. In Section 4 we introduce a framework for the statistical analysis, and present various approaches to the estimation problems. Some results from river Gaula are shown in Section 5, while Section 6 gives a brief summary and some conclusions.

2. River Gaula – Some Background

This section gives a brief presentation of river Gaula and its watershed, with special emphasis on the factors that affect the contamination in the river.


2.1. Geography and Hydrology
River Gaula is the largest river in Central Norway when measured by the area of its watershed: 3653 km². The geographic location of the river and its watershed is shown in Figure 1. The length of the river is about 150 km, and it descends 800–900 m from source to mouth. There is approximately 6000 lakes of various sizes in the watershed. The lakes cover only 2.7% of the area, which is little compared to most rivers in Norway.

There are large variations in the runoff in Gaula, due to the climate, topography and the low percentage of lakes in the watershed. At Støren, in the lower part of the river, the mean runoff is 76.5 m³ s⁻¹, with an observed minimum of 2.2 m³ s⁻¹ and a maximum of 3060 m³ s⁻¹ during a disastrous flood in August 1940.

Fig. 1. Location of river Gaula and its watershed.

2.2. Fishing and Recreation
Some of the most scenic parts of Norway may be found along Gaula. There are vast unspoiled areas of forests and mountains, and about 1500 private cabins. The lower parts of Gaula are among the most important salmon rivers in Norway, and the tributaries are excellent for trout and char. Due to the pollution from the old mines, described below, no fish can survive in the upper parts of the main river.

Due to its recreational qualities, some years ago Gaula was declared a conservation area by the Norwegian Parliament. This prohibits any further exploitation of the river for commercial purposes.

2.3. Mining
Kjøli and Killingdal copper mines are located near the head of the river. They were in operation for centuries, but are now closed. Kjøli has not been operating since 1941, while Killingdal was closed only recently – in 1986. The waste from the earlier
production, however, still spills into the river, leading to serious contamination by heavy metals (Cu, Zn, Ca, Fe) and sulphates. No fish can survive in the most severely affected areas.

3. Monitoring Program

After Gaula was declared a conservation area, more concern was given to the pollution originating from the mines in the upper part as well as from human and agricultural sources further down the river. In 1986, the National Program for Pollution Monitoring thus started an investigation of Gaula. The main goal is to: ‘Describe the situation in the river so that the need for action may be evaluated, and to identify and quantify point source pollution, to decide where and what kind of actions that may be most effective’. The investigation will be finished by the end of 1988.

The program is organized and carried out by the Norwegian Water Research Institute, in collaboration with local authorities. The Norwegian Computing Center is engaged to assist in the statistical analysis and the presentation of the results.

3.1. Stations, Measurements and Observation Frequency

The measurements in the program are taken at 10 stations along the main river, and at 11 stations near the mouth of various affluent rivers. The locations of the stations on the main river are shown in Figure 2.

Fig. 2. Measurement stations along the main river.
The choice of variables to be observed at each station was made according to the nature of the sources of pollution. In the upper parts, where the mines and their effect is the main concern, most emphasis was put on the concentration of heavy metals and pH. Further down, where human and agricultural sources are the most important, the measurements were concentrated around phosphorus, nitrogen and nitrates, combined with counts of coliform bacteria, etc.

The measurements were started in June 1986, and for this study we had observations available up to mid-December 1987. The number of observations was between 33 and 47 at the six uppermost stations. Further downstream less measurements were taken. The observations are taken at irregular intervals. During the two summer periods there are 3–4 observations per month, while there are very few, if any, during the winter 86-87 and early spring 1987.

The runoff is measured continuously at two (three from 1987) stations along the main river.

3.2. Scope of the Norwegian Computing Center’s Work

The goal of the Norwegian Computing Center’s contribution to the project is, among others:

- To estimate various statistical characteristics for concentration and loadings along the river.
- To quantify the effect of the contamination from the copper mines both locally and further downstream.

In the present paper we will concentrate only on the analysis of the copper contamination. We present the methodology, and give some results.

4. The Problem and Some Possible Solutions

In this section we present a statistical framework for the subsequent analysis, define the problem and discuss some possible solutions.

4.1. Notation and the Problem

Let \( c(t) \) denote the concentration of some variable of interest, Cu say, at time \( t \). In general, \( c(t) \) will vary continuously over time. Figure 3 illustrates a (hypothetic) possible course of the concentration over a year. Normally, \( c(t) \) cannot be continuously observed, but only sampled at certain points in time.

It is possible to do some analysis within this continuous time framework. However, for practical purposes it is more convenient to discretize the process, defining the discrete process \( c_n \) as the mean concentration on day \( n \), where the days are numbered sequentially during the period of interest. In this study we have used days as the basic time resolution, other interval lengths (hour, pentad, week, month) are of course possible. A further refinement is to define \( c_n \) as the mean concentration weighted according to the runoff. \( c_n \) then denotes the mean concentration in the water passing during the day, and not the mean concentration over time. With a one day
time resolution the difference may be of minor importance, but for longer periods the effect may be substantial.

We thus have:

\[ c_t = \sum_{s \text{ in day } t} c(s)ds \text{ (unweighted)} \]

or

\[ c_t = \sum_{s \text{ in day } t} c(s)q(s)ds/q_i \text{ (weighted)} \]

where \( q(t) \) denotes the runoff at (continuous) time \( t \) and \( q_i \) is the mean runoff during day no. \( i \), given by:

\[ q_i = \sum_{s \text{ in day } i} q(s)ds. \]

\( c_t \) is actually observed only for a few days in the period of interest. Based on these observations, we are concerned with questions like:

- What was the mean and/or median concentration during the period of interest?
- What is the upper p-percentile (95% say) in the distribution of the concentration?
- What was the average and total loading during the period?

Note that we are only concerned with what actually happened during the period of interest – not with the parameters of some underlying theoretical model and/or process.

4.2. Solution with a sampling framework

One approach to the above problems is through the theory of random sampling. Here it is assumed that each day has a fixed value \( c_t \), and an associated random variable \( I_t \), given by
\[ I_i = \begin{cases} 
1 & \text{if } c_i \text{ is actually observed} \\
0 & \text{otherwise} 
\end{cases} \]

One basic assumption in this approach is that there is no connection between \( I_i \) and \( c_i \); the probability that a day is actually sampled does not depend on the value of \( c_i \). If this assumption is fulfilled, one can estimate the mean, median and other characteristics of the 'population', that is the total period of interest, by the similar characteristic of the sample. The mean will be unbiased, other characteristics may be more or less biased according to the (unknown) actual values of all the \( c_i \). This assumption is not necessarily fulfilled in environmental studies, as discussed below.

The standard deviation of the estimates may also be computed, assuming various probability mechanisms to control the sampling, i.e. the \( I_i \) process. In the simplest case, when the days are sampled independently with a certain probability, the formula for the standard deviation of the mean estimate becomes fairly simple, and can be found in any text book on sampling. When the mechanism becomes more complex, so does this formula. The standard deviation of other characteristics are not easily obtained even in the simple, random sampling case, and quickly becomes untractable for more complex sampling patterns.

The assumption about independence between the sampling and the actual values of the process under study is frequently violated in environmental studies. Periods of interest, e.g. during heavy rainfall, are often sampled more frequently. Taking simple averages may then give too much weight to extreme values, introducing bias in the estimates. In addition, upper Gaula is inaccessible when there is a heavy snow cover, which prohibits frequent sampling during the winter months. Since the concentration, in general, is lower during the winter, this results in a somewhat biased sampling. This may, to a certain extent, be adjusted for taking weighted averages, where the observations are weighted according to the length of the time gap between each observation and the previous one.

A similar time-weighting technique can be applied when the objective is to estimate the median or other percentiles. To give each observation a weight according to the number of days since the previous one is equivalent to constructing an extended sample by copying each observation \( D_i \) times, where \( D_i \) is the number of days since the previous observation. By sorting this extended sample, a time-weighted estimate of the median or other characteristics may be obtained.

Another problem arises when there is seasonal variation in the process, for example a year cycle. If the period of observation is not an integer number of cycles, this may introduce a bias if the undersampled period differs significantly from the overall mean. This may be avoided using a two step procedure: Let \( P \) be the number of complete cycles (e.g. years) covered by our observations. First the (possibly time weighted) mean is computed from the observations taken during the first \( P \) cycles, ignoring the redundant observations at the end. Then the mean is computed for the observations taken during the last \( P \) cycles, ignoring the redundant observations in the beginning of the series. Finally, the two means thus obtained are averaged to
obtain a seasonally corrected estimate. The same approach may be used for estimating the median or other percentiles, possibly combined with time-weighting.

It is in general extremely difficult, if at all possible, to find exact analytical expressions for the variance of the adjusted estimators described above, unless very restrictive assumptions are made for the sampling mechanism and the relationship between sampling and process.

Let us illustrate the issues considered above with an example. Table I shows the observed Cu concentrations (μg 1⁻¹) at Station 6 during the period 860610–871208, 40 observations in all. The dates and the time (in days) since the previous observation are also given. The average time between observations is 14 days, with a range [1,35].

**TABLE I**

<table>
<thead>
<tr>
<th>Date</th>
<th>Cu</th>
<th>( D_j )</th>
<th>Date</th>
<th>Cu</th>
<th>( D_j )</th>
<th>Date</th>
<th>Cu</th>
<th>( D_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>860610</td>
<td>15.6</td>
<td>(14)</td>
<td>860929</td>
<td>43.5</td>
<td>3</td>
<td>870630</td>
<td>20.1</td>
<td>13</td>
</tr>
<tr>
<td>860623</td>
<td>17.5</td>
<td>13</td>
<td>861013</td>
<td>36.5</td>
<td>14</td>
<td>870701</td>
<td>19.5</td>
<td>1</td>
</tr>
<tr>
<td>860625</td>
<td>14.5</td>
<td>2</td>
<td>861021</td>
<td>35.5</td>
<td>8</td>
<td>870714</td>
<td>22.0</td>
<td>13</td>
</tr>
<tr>
<td>860708</td>
<td>31.5</td>
<td>13</td>
<td>861112</td>
<td>29.0</td>
<td>22</td>
<td>870721</td>
<td>16.0</td>
<td>7</td>
</tr>
<tr>
<td>860710</td>
<td>16.0</td>
<td>2</td>
<td>861209</td>
<td>22.0</td>
<td>27</td>
<td>870804</td>
<td>19.0</td>
<td>14</td>
</tr>
<tr>
<td>860729</td>
<td>4.8</td>
<td>19</td>
<td>870113</td>
<td>3.4</td>
<td>35</td>
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<td>860812</td>
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<td>14</td>
<td>870213</td>
<td>18.0</td>
<td>31</td>
<td>870901</td>
<td>31.0</td>
<td>14</td>
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<tr>
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<td>7</td>
<td>870317</td>
<td>6.4</td>
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<td>870915</td>
<td>44.0</td>
<td>14</td>
</tr>
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<td>1</td>
<td>870407</td>
<td>14.0</td>
<td>21</td>
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<td>31.5</td>
<td>14</td>
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<td>860826</td>
<td>17.0</td>
<td>6</td>
<td>870504</td>
<td>21.0</td>
<td>27</td>
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<td>14</td>
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<td>860909</td>
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<td>14</td>
<td>870526</td>
<td>60.0</td>
<td>22</td>
<td>871110</td>
<td>27.3</td>
<td>28</td>
</tr>
<tr>
<td>860916</td>
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<td>7</td>
<td>870602</td>
<td>25.0</td>
<td>7</td>
<td>871208</td>
<td>15.5</td>
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<td>36.0</td>
<td>7</td>
<td>870616</td>
<td>17.5</td>
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<td></td>
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</tr>
<tr>
<td>860926</td>
<td>29.5</td>
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<td>870617</td>
<td>24.5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II shows the mean, the median and the upper 95% percentile obtained from the observations alone with no adjustments, from time-weighting alone, seasonal correction alone and the two combined.

In the time-weighting, each observation is given a weight \( D_j \) according to the number of days since the previous observation, as given in Table I. For the first observation, where \( D_j \) is unknown, the average interobservation period of 14 days is used.

The seasonal correction is performed by first calculating the various characteristics for the 26 observations taken during the 12-months period 860610–870609, with or without time-weighting. Similarly, the characteristics for the 22 observations taken during the 12-months period 861209–871208 are found, and the two are averaged. In this case, the number of observations during the two periods are very similar. If this had not been so, different weights might be given to the statistic for each of the two periods in the final averaging.
TABLE II
Resulting characteristics for the Cu concentration (μg l⁻¹) at Station 6 when different methods of adjustment are applied

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Methods of adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
</tr>
<tr>
<td>Mean</td>
<td>25.1</td>
</tr>
<tr>
<td>Median</td>
<td>23.0</td>
</tr>
<tr>
<td>95%</td>
<td>59.3</td>
</tr>
</tbody>
</table>

It is clear from Table II that the time-weighting effects the estimates of the mean significantly, while the seasonal adjustment is of less importance. The median and the 95% percentile are much less affected by the various adjustment methods.

The analysis for the loadings follows a similar pattern as above. The daily loadings \( z_i \) may be computed as \( z_i = c_i q_i \) (concentration times runoff), and the mean, median and other characteristics estimated using the same procedures, with or without weighting. The runoff, however, will often be observed continuously, and we may obtain a better estimate, taking the additional information into account. An adjusted estimate of the mean loading is given by:

\[
\bar{z} = \bar{z} \cdot \bar{q} / \bar{q}.
\]

Here \( \bar{z} \) is the mean loading based on the observed days only, with or without time-weighting and seasonal correction. Similarly, \( \bar{q} \) denotes the mean runoff based on the days with observed concentrations only, using the same weighting and correction as for \( z_i \), if any. Finally, \( \bar{q} \) is the mean runoff based on all the days in the period. The estimator \( \bar{z} \) is thus corrected to be consistent with the total runoff. The estimator \( \bar{z} \) is in general better than the unadjusted \( z \), especially when concentration and runoff are positively correlated. The gain in precision, as measured by the reduction in standard deviation, can be considerable (Lingsten and Sæbo, 1987). For the Cu loading at Station 6, this method (without time-weighting or seasonal corrections) gives a mean loading of 585 mg s⁻¹ with a standard deviation of 106.

4.3. Solution using a time series model for the process

The above is about as far as one can get without any further assumptions on the stochastic mechanism that generates the \( c_i \). In this section we present a simple model which gives a satisfactory description of this mechanism, and shows how this concept may be used to construct other estimates and to study their precision.

The main idea is to construct a stochastic model and to estimate its parameters. Using this model and the actually observed concentrations, we can construct optimal estimates for the remaining, unobserved \( c_i \) values, and combine estimates with the observations to compute the various characteristics.
4.3.1. The Dynamic Linear Model

The model is very simple. We assume that the concentration $c_t$ varies randomly around a slowly changing mean $\mu_t$, where the evolution of the mean can be described by a random walk. Formally:

$$
\mu_t = \mu_{t-1} + \omega_t \\
\log y_t = \mu_t + \nu_t
$$

Here $\mu_t$ denotes the smooth level of the process, and $\omega_t$ is the random change in level from one day to the next. $y_t$ is the logarithm of the observed concentration (log $C_t$), which may be decomposed into the level $\mu_t$ and a term $\nu_t$ which is the deviation from the level at time $t$. $\omega_t$ and $\nu_t$ are the noise terms in the model, and they are assumed to be independent, identically normally distributed with mean zero and variances $W$ and $V$ respectively. Note that we actually model the log-transformed series of concentrations. This is because concentration data are notoriously skewed with occasional extremely large values, which are not consistent with a normal distribution. To reduce the effect of these values, and have more normally distributed data, the logarithmic transform is applied.

One possible justification for this model is the following: In the previous section we have assumed that the observed $c_t$ is the mean concentration for the relevant 24 hours period. If the analysis is based on water samples collected continuously during this period, this assumption will be fulfilled. In most cases, including the Gaula program, the sample is taken at one instant only, by filling a bottle in the river. This instantaneous observation will of course differ from the (unobserved) daily mean. In addition, there may be errors in the laboratory analysis. One interpretation of the model framework described above is then to regard $\mu_t$ as the unobserved daily mean concentration (on a log-scale) which changes by a amount $\omega_t$ from day to day, while $\nu_t$ is the amount which the instantaneous sampled concentration differs from the daily mean, plus analysis errors etc.

This justification is not necessarily correct, however, and other explanations may be possible. In general, $\mu_t$ represents the systematic variation in the series, while $\nu_t$ constitutes the random part.

Other time series models, e.g. within the ARIMA framework, could be applied. Unfortunately, the irregular spacing of the observed values creates difficulties, as most time series models are developed for equispaced data. There are ways to get around this problem see e.g. Jones (1985) or Kohn and Ansley (1986). However, the present formulation falls directly within the framework of Dynamic Models (DLMs) (Harrison and Stevens, 1976) and lends itself to calculations using the Kalman filter. This approach is just as applicable for irregularly spaced data. We have used one day as the basic time unit, and regard the unobserved data as missing. Thus, we have a time series with more than 90% missing data.
Fig. 4. Interpolated Cu-concentrations at Station 6, using various combinations of the variance parameters in the DLM. (a): $W = 0$, (b): $V = 0$, (c): $W = 0.01$, $V = 0.12$, which are the ML-estimates.

4.3.2. Interpolation

The variance parameters $W$ and $V$ can be estimated from the data using maximum likelihood (ML). Given the model and the observations, we can obtain optimal estimates for the missing values in the series, again using the Kalman filter technique. These optimal estimates are given by the estimated $\mu_t$, the process means.
The model can give rise to various forms of interpolated values, according to the values of the variances $W$ and $V$. Figure 4 presents three variations, all for the Cu concentration at Station 6. Panel (a) shows the interpolated values obtained when $W$ is 0. In this case there is no variation in the $\mu_t$ sequence, and the observations vary around a constant mean. Panel (b) gives the interpolated values when $V$ is 0. Then $\mu_t = \ln(c_t)$ whenever the concentration is observed, and the optimal interpolated values are straight lines (in the log-scale) between the observations. Finally, panel (c) shows the interpolated sequence for the ML-estimates of the variances, which in this case are 0.01 and 0.12 for $W$ and $V$ respectively. The result is a trade-off between the two extremes given in (a) and (b).

Given this optimal set of interpolated values, it is straightforward to obtain estimates of the mean concentration:

$$\bar{c} = \frac{1}{T} \sum_{t=1}^{T} \exp(\mu'_t)$$

or the mean concentration weighted according to the runoff:

$$\bar{c} = \frac{1}{T} \sum_{t=1}^{T} q_t \cdot \exp(\mu'_t) / \bar{q}$$

or the mean loading:

$$\bar{z} = \frac{1}{T} \sum_{t=1}^{T} q_t \cdot \exp(\mu'_t)$$

Above, $T$ is the length of the period of interest. $\mu'_t$ denotes the interpolated value at day $t$, $q_t$ is the observed runoff at day $t$ and $\bar{q}$ is the mean runoff during the period.

The $[\mu'_t]$ are unbiased estimates of the true $[\mu_t]$ sequence. Unfortunately $\exp(\mu'_t)$ is no longer unbiased for $\exp(\mu_t)$, due to the well known properties of the log-normal distribution. Our estimates, $\exp(\mu'_t)$, will tend to be slightly too low, compared with the true values. It is possible to adjust the results, taking this point into consideration. However, this has not yet been done.

As estimates of the true $\mu_t$, the $\mu'_t$ have certain variances, which are given by the Kalman filter recursions as well. These variances will change with time – in general they will be smaller the nearer we are to an actual observation. This may be utilized in the formulae above, weighting the terms in the sums according to the inverse of their variances.

A philosophical question arises: On the days where the concentration is actually observed, should the observed $c_t$ or the estimated $\mu'_t$ be used in the formulae above? There is no obvious answer. If one believes that the observed values are 'true', so that the $\nu_t$ contains the part of $y_t$ which is non-systematic, but never the less true, then the observed values should be used. On the other hand, if one regards the $\nu_t$
as deviations from the daily mean due to sampling and analysis errors, then $\mu_i$ would be right. In our situation, with less than 10% of the days observed, the effect on the results is negligible.

The resulting estimates, using these variations of the interpolation method, are shown in Table III.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Using $\gamma_i$</th>
<th>Using $\mu_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean concentration ($\mu g \text{ l}^{-1}$)</td>
<td>21.8</td>
<td>21.7</td>
</tr>
<tr>
<td>Mean concentration, weighted by runoff ($\mu g \text{ l}^{-1}$)</td>
<td>26.7</td>
<td>26.3</td>
</tr>
<tr>
<td>Mean loading ($\mu g \text{ s}^{-1}$)</td>
<td>517</td>
<td>510</td>
</tr>
</tbody>
</table>

The $\mu_i$-estimates $\mu_i^*$ are optimal and unbiased. However, the sequence of estimates, $\{\mu_i^*\}$, will have a much too smooth appearance compared to the true $\{\mu_i\}$, not to mention $\{\gamma_i\}$, since the random effects are not taken into consideration. Thus, estimates of variances and percentiles cannot be made from the $\mu_i^*$-estimates, and it is difficult to assess the precision of the various means shown above. To resolve this problem we have to turn to simulation, as described in the next section.

4.3.3. Simulation

For a given pair of variances, the DLM can be used to generate a number of independent possible outcomes of the $\gamma_i$ process. Each realization will be consistent with the actually observed values, and the distribution of the generated values will be the same as for the observation, assuming the model to be correct.

From each generated outcome, the (various) means, medians, ranges and percentiles can be calculated. Final estimates if these characteristics can then be obtained as the average over all the simulations, and the variance of the estimates can be found from the variability between the simulations.

In Figure 5 panels (a) and (b) show two simulated realizations of the Cu concentration at Station 6. Both the series pass through the observed values, and have a rather similar overall appearance. Individual values, however, may differ substantially. Panel (c) shows the mean, maximum and minimum taken over 200 simulations. The mean is very similar to the estimated $\mu_i$-sequence presented in Figure 4(c), as expected, and the range is considerable, especially at times far from any observation, which obviously makes sense.

Using this technique, Table IV presents some estimated characteristics for Station 6, with their standard deviation.

Note that the means taken over the 200 simulations are slightly higher than the values reported in Table III, in accordance with the discussion in the previous section.
Fig. 5. (a) and (b): Two simulated realizations of the Cu-concentration at Station 6. (c): Mean, maximum and minimum based on 200 simulations.
TABLE IV

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>St. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean concentration (µg l⁻¹)</td>
<td>23.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Median concentration (µg l⁻¹)</td>
<td>20.7</td>
<td>1.5</td>
</tr>
<tr>
<td>95% percentile for the concen. (µg l⁻¹)</td>
<td>52.2</td>
<td>4.8</td>
</tr>
<tr>
<td>Mean loading (µg s⁻¹)</td>
<td>562.</td>
<td>45.</td>
</tr>
</tbody>
</table>

4.3.4. Comparison with the Sampling Approach

Similar analysis applied to various chemicals at various stations along the river gives a number of estimates, with standard errors. These can be compared to the results obtained by using the simple, random sampling (SRS) approach described earlier. In most of the cases the simulation method outperforms the SRS, giving 20–50% reduction in the variance. However, the SRS-estimates for the variance usually provides an upper limit for the real sampling variance. This is because the actual observations usually will be more evenly distributed in time compared to a SRS scheme. There are also a few cases where the variance from the simulations becomes larger than those from SRS, so the conclusion is not quite clear. But an obvious advantage of the simulation technique is that it provides measures of the precision for all kinds of characteristics, as opposed to the simple, random sampling approach. On the other hand, the simulation technique is more complex, and more demanding as far as software and computer resources are concerned. With the present state of computer technology, however, this is no longer any serious objection.

5. Results

It is beyond the scope of this paper to present our findings in detail. The final report from the monitoring program, will contain all the details. A more thorough treatment of the various statistical aspects may be found in Sæbbe (1987) and Aldrin (1988a, 1988b). This section presents some results on the heavy contamination from the mines in the upper part of the river.

The mines are located just above Station 2 (Kjøli) and Station 3 (Killingdal). The ore at both mines contains copper, while Killingdal is rich on zinc as well. Figure 6 presents the estimated median concentration of Cu and Zn for the six uppermost stations in the river, and Figure 7 gives the mean loading for the same stations. The net loadings for the sections between the stations can also be calculated, and they are shown in Figure 8. The impact of both the mines on the Cu-loadings, and of Killingdal on the Zn-loadings, is clearly illustrated.
Fig. 6. Median concentration of Cu and Zn for the six uppermost stations along the river.

Fig. 7. Mean loadings of Cu and Zn for the six uppermost stations along the river.

Fig. 8. Net loadings of Cu (left column, SW-NE shading) and Zn (right column, SE-NW shading) between each of the six uppermost stations along the river.
The net loading at Station 2 have been compared to a few observations directly from the spill from the Kjøli mines, and the results coincides (Sebø, 1987).

6. Summary and Conclusions

We have given a brief presentation of the Gaula monitoring program, with special emphasis on the heavy metal pollution from the mines at the upper part of the river.

We then presented two techniques to estimate various characteristics of the concentration and loading, if possible with their standard deviations. The first is based on the concept of simple random sampling from a finite population, while the other is based on an underlying Dynamic Linear Model for the process under study, and use of the Kalman filter.

The first is simple to use to estimate means and other simple characteristics, but is generally inapplicable for more complex characteristics. We used this technique to estimate mean, median and the 95% percentile of the Cu concentration at one measurement station. We demonstrated the effect of adjusting the estimates by time-weighting and seasonal correction. We presented an estimate of the mean loading, taking into account the additional information available if the runoff are observed continuously.

The second approach is more complicated, but provides all kinds of characteristics, as well as estimates of their precision. The first step of this technique was to estimate the parameters in a simple time series model describing the underlying Cu process. Then 200 possible realizations of the real Cu process were simulated from the model. By averaging over the simulations, various characteristics could be calculated, with standard deviations estimated on the basis of the variability between the simulations.

The paper finally shows how the techniques can be applied to obtain quantitative information of the impact of the point sources, namely the old mines, on the loading.

Acknowledgement

The authors appreciate the help, encouragement and valuable discussions with the scientists at the Norwegian Water Research Institute, in particular Tor S. Traaen, who has been in charge of the project. We are also grateful to our colleague Sigmund Kalvenes for his efforts both during the project work and during the preparation of this paper.

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