Optimal rebalancing of portfolios with transaction costs assuming constant risk aversion

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OPTIMAL REBALANCING OF PORTFOLIOS WITH TRANSACTION COSTS ASSUMING CONSTANT RISK AVERSION

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Abstract. The paper describes how to rebalance a portfolio with transaction costs, assuming constant risk aversion. The transaction costs reduce the capital. This is included in the utility function, ensuring a consistent trade off between the transaction costs and the cost of deviating from the optimal portfolio. There is a no-trade region for the relative weights where it is optimal not to trade. For proportional transaction costs, it is optimal to rebalance to the boundary when outside the no-trade region. With flat transaction costs, one should rebalance to a state on an internal surface in this region and never perform a full rebalance. The no-trade region is approximated by two parameters for each pairs of assets. The paper discusses the case with n symmetric assets and general stochastic processes. The results are illustrated by an example with 5 assets where we obtain 75% reduction in transaction costs.

1. Introduction

An important part of portfolio management is to find the optimal weights of the different assets, and rebalancing according to these. Transactions costs are often neglected although these influence the optimal strategy in an essential manner as documented in Atkinson *et al.* [1], Donohue and Yip [5] and Leland [7]. Most papers conclude that there is a no-trade region for the relative weights where trade should not be performed. When the weights are outside this region, one should rebalance to its boundary if the transaction costs are proportional. If the transaction costs have fixed or flat elements, one should rebalance to a state on an internal surface in the no-trade region but never perform a full rebalance. We believe this was first proved in Holden and Holden [6], assuming only a concave and smooth utility function. This paper describes the optimal rebalance strategy assuming constant risk aversions. We include the effect of correlation between the different assets, the asymmetry of selling and buying the different assets, in addition to all kinds of transaction costs.

Despite the theoretical agreement that a no-trade region exists, it is not common practice to take this into consideration. Donohue and Yip [5] state that an optimal rebalance strategy typically reduces transaction costs by 50% . The two examples in Holden and Holden [6] and the example in this paper give 75% reduction. Most portfolio managers rebalance to what they considered is the optimal balance at fixed time intervals, often monthly or quarterly, see e.g. Leland [7]. Others define intervals for the weight of each asset, adjusting to the boundary of these intervals either regularly or continuously. Frequently, these decision criteria are combined with a full rebalance in certain situations.

The transaction costs reduce the capital that may be invested later. This is included in the utility function used in this paper. Hence, we are able to make a

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consistent trade off between the transaction cost and the cost of deviating from the optimal portfolio as opposed to the model applied in Sun *et al.* [12] and Leland [7]. The no-trade region is approximated by two parameters for each combination of two assets. These parameters describe when there should be a rebalance between the two assets. The example shows that this flexibility is necessary since the boundary of the no-trade region depends heavily on the parameters for each pair of assets.

This paper focuses on constant risk aversion. Liu [8] gives a thorough discussion of this problem with one risk-free investment and n uncorrelated geometric Brownian motion investments. All rebalance is between an investment with risk and the risk-free investment. Moreover, the no-trade region is a fixed threshold for each investment with risk. In this paper, all assets are treated in the same manner. Hence, a rebalance may be between any combinations of assets.

Dybvig [4], in a non-published paper, describes the no-trade region analytically with a mean-variance utility function with n symmetric assets. A disadvantage with this approach is that the no-trade region is symmetric in the high and low expectation assets. With constant risk aversion there is an asymmetry making it more likely to not rebalance when a high expectation asset has a low relative weight since it is more likely that this asset increases in value and hence also increases in relative weight without a rebalance. This is shown in Tables 2. But with constant risk aversion there is no analytic description of the no-trade region. We choose an approximation to the no-trade region with a parameterization that may describe the analytic no-trade region with a mean-variance utility function.

One of the first papers to study portfolio optimizations with transaction costs was [3]. The case of two assets is analyzed analytically by Taksar *et al.* [13] and Øksendal and Sulem [11]. The multi-asset problem under strong assumptions has been studied by Donohue and Yip [5]. See also Chang [2] and Liu [8]. Sun *et al.* [12] and Leland [7] use dynamic programming algorithms to determine the no-trade region in higher dimensions. Atkinson *et al.* [1] give an approximate formula for the boundary.

Section 2 describes the model and its theoretical properties and is followed by a section with some general advice regarding rebalancing. Section 4 illustrates how to find the optimal relative weights and the no-trade region. We simulate realizations with four different strategies in a case with $n = 5$ assets. This example shows that the transaction costs are reduced by 75% using an optimal no-trade region compared to monthly rebalance when the transaction costs are proportional. The paper is ended by some closing remarks in Section 5.

2. The model

In our model, the transaction costs reduce the capital that may be invested later. This ensures a consistent trade off between transaction costs and the need to rebalance to a more optimal portfolio. Consider n assets, and let $V_{i,t}$ denote the stochastic value of asset i, for $i = 1, 2, \ldots, n$, at time t. We assume that the stochastic properties of $V_{i,t}$ are known, and that the relative increments are stationary, e.g. $V_{i,s}/V_{i,t}$ is only a function of $s - t$.

Let the value of the portfolio at time t be

$$W_t = \sum_{i=1}^{n} a_{i,t} V_{i,t}, \tag{1}$$

where the weight of the assets at time t is $\mathbf{a}_t = (a_{1,t}, a_{2,t}, \ldots, a_{n,t})$. The relative weights are defined by

$$w_{j,t} = \frac{a_{j,t} V_{j,t}}{W_t}. \tag{2}$$

With constant risk aversion γ and an infinite time horizon, we want to optimize the utility function

$$U(W_t) = \frac{1}{1-\gamma} \int_t^\infty E\{W_s^{1-\gamma}\} \exp(-\beta s) ds \tag{3}$$

at any time t. When $\gamma > 0$, small values of W have more influence on the utility than large values. Then it become more important to avoid low values of W than to obtain large values. We typically have $\gamma > 1$, implying a negative utility function. The β term expresses the discountation. When comparing different utility values, we will use the corresponding risk free interest rate

$$r_U = \exp(\beta/(1-\gamma) - 1/(U(1-\gamma)^2))$$

giving a better intuition for the values. The risk free interest rate r_U gives the utility U.

The investor may rebalance the portfolio at any time. We will consider only one rebalance at a time. A rebalance at time t implies that the portfolio is changed from

$$W_{t-} = \sum_{i=1}^{n} a_{i,t-} V_{i,t} \tag{4}$$

to

$$W_{t+} = \sum_{i=1}^{n} a_{i,t+} V_{i,t}, \tag{5}$$

where $a_{i,t}$ are functions of time t that are constant between each rebalancing, and $a_{i,t-}$ and $a_{i,t+}$ are the values before and after the rebalance, respectively. In case there is a jump in the value of the assets, $V_{i,t}$, we will always let $V_{i,t}$ denote the values after the jump. If a jump in $V_{i,t}$ implies a rebalance, this is performed immediately, and obviously based on the values after the jump, i.e., $V_{i,t} = V_{i,t+}$. We allow the investor to rebalance at any time, but in practice this may be the end of each trading day. A rebalance may include transaction costs. In this paper we consider proportional transaction costs at a time t is defined as

$$c_{t,P} = \sum_{i=1}^{n} c_{i,P} |a_{i,t+} - a_{i,t-}| V_{i,t} \tag{6}$$

and flat transaction costs defined as

$$c_{t,F} = \sum_{i=1}^{n} c_{i,F} \delta_i \tag{7}$$

where $\delta_i = 1$ if there is transaction in asset i at time t and $\delta_i = 0$ otherwise. These two types cover the properties of interest from a theoretical point of view. Moreover, it is trivial to extend such that the transaction costs, e.g., depend on whether the investor sell or buy an asset. The transaction costs at time t reduces the value of the portfolio i.e.

$$W_{t+} = W_{t-} - c_{t,P} - c_{t,F}. \tag{8}$$

At every time t, the investor optimizes $U(W_t)$. The simplest possible strategy is never to rebalance, that is, $a_{i,t}$ is constant in time. Another strategy is to rebalance to some ideal relative weights

$$w_{i,t+} = \tilde{w}_i$$

for $i = 1, 2, \cdots, n$ for the given times t e.g. at the end of each month, quarter or year. This is denoted full rebalance in the introduction. A third strategy is rebalance in order to have the relative weights within a given interval, thus

$$\hat{w}_i^1 \leq w_{i,t+} \leq \hat{w}_i^2$$

for $i = 1, 2, \ldots, n$. This strategy requires detailed rules to determine how to rebalance. For instance, rebalance only when a relative weight is outside the boundary of the admissible interval and in this case, rebalance to the assets with weights closest to the opposite boundary. Other strategies are of course also possible, such as the one we argue for in Section 4.

Holden and Holden, [6], prove that for any concave and continuously differentiable utility function including (3):

- If there are no transaction costs, there are some optimal relative weights, $\tilde{\mathbf{w}} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$, and it is optimal to always rebalance to these.
- If there are transaction costs, i.e. we do not have $c_{i,P} = c_{i,F} = 0$ for all i, there is a no-trade region R_t with $\tilde{\mathbf{w}}$ in R_t, where it is optimal not to trade.
- If there are only proportional transaction costs, i.e. $c_{i,P} > 0$ for some i, there is a no-trade region R for the relative weights that is independent of time. When relative weights, $\mathbf{w}_t$ are outside R, it is optimal to rebalance to the boundary of R.
- If there are flat transaction costs, i.e. $c_{i,F} > 0$ for some i and independent on whether there are proportional transaction costs, then the no-trade region R_t also depends on the value W_t of the total portfolio. When relative weights, $\mathbf{w}_t$ are outside R_t, it is optimal to rebalance to an internal surface in R_t.

3. Some general advice

There is a balance between a strategy with a small no-trade region with high transaction costs, and a strategy with a larger no-trade region, but the transaction costs are lower. If there is a small no-trade region, the relative weights are close to the optimal $\tilde{\mathbf{w}}$, but the transaction costs are high.

The optimal relative weights $\tilde{\mathbf{w}}$ and the no-trade region are sensitive to small changes in the parameters of the stochastic process of the assets. Hence, if there is uncertainty in these parameters, it should be included in the modeling. This entails a larger no-trade region avoiding a rebalancing that we later regret.

An increase or decrease of the investment is an opportunity to rebalance to a lower additional cost. If such changes are known in advance, the size of the no-trade region will increase when approaching the time of the change. Actually, it may be optimal to rebalance only when we increase or decrease the total investment.

We may expect to sell assets with high expectation regularly in order to maintain the relative weights. However, if these assets have decreased in value, it may be optimal to wait for an increase, in which case a rebalancing will not be necessary. This asymmetry is shown in Tables 2 with $P > 0$ for some of the asset pairs. The significance of this effect depends on the difference in expectation and variability.

4. Example

In this section, we illustrate the theory in an example with $n = 5$. First, we find the optimal relative weights $\tilde{\mathbf{w}}$ when there are no transaction costs. If we continuously rebalance to $\tilde{\mathbf{w}}$, it is easy to prove that $E\{W_t^{1-\gamma}\}$ changes exponentially in

i	μ_i	σ_i	$\rho_{i,1}$	$\rho_{i,2}$	$\rho_{i,3}$	$\rho_{i,4}$	$\rho_{i,6}$	$\tilde{w}_{i,\gamma=3}$	$\tilde{w}_{i,\gamma=4}$	$\tilde{w}_{i,\gamma=6}$
1	1.1	0.22	1	0.7	0.1	0.3	0.1	0.333	0.249	0.155
2	1.09	0.20	0.7	1	0.05	0.1	0.2	0.223	0.170	0.120
3	1.05	0.12	0.1	0.05	1	0	0	0.236	0.198	0.156
4	1.035	0.04	0.3	0.1	0	1	0.3	0.016	0.064	0.210
5	1.035	0.04	0.1	0.2	0	0.3	1	0.188	0.318	0.359

TABLE 1. Expectation μ_i, standard deviation σ_i, correlation $\rho_{i,j}$ and optimal relative weights $\tilde{w}_{i,\gamma}$ for each asset i for three different values of γ and $n = 5$. It is assumed $V_{i,t}$ are geometric Brownian motions.

time. We find $\tilde{\mathbf{w}}$ by optimizing the expression

$$E\{W_{\Delta t}^{1-\gamma}\} = E\{(\sum_{i=1}^{n} \tilde{w}_i V_{i,\Delta t})^{1-\gamma}\} \tag{9}$$

for a small time step Δt where we have normalized $W_0 = V_{i,0} = 1$. The optimal values may be found by any optimizer. It is difficult to find a good analytic approximation to the optimal relative weights $\tilde{\mathbf{w}}$ since these are very sensitive to small changes in the parameters. An approximation may affect the optimal relative weights substantially, but not necessarily the utility.

Table 1 shows the parameters for the five assets that are assumed representative for the Norwegian stock marked, international stock marked, Norwegian real estate, Norwegian bonds, and international bonds. It also exhibits the optimal relative weights $\tilde{\mathbf{w}}$ for the five assets for three different values of γ.

Assume first there are only proportional transaction costs, i.e. $c_{k,P} > 0$ and $c_{k,F} = 0$ for all k. We do not know the exact form of the no-trade region. It is natural to rebalance when the relative weights $w_{i,t}$ and $w_{j,t}$ of two assets are high and low respectively, compared to the optimal values $\tilde{w}_i$ and $\tilde{w}_j$. A rebalance that involves more than two assets, may be split into several independent rebalancings, each only involving two assets. This makes it natural to approximate the no-trade region by

$$R_P = \{\mathbf{w}_t \mid -D_{j,i} < w_{i,t} - w_{j,t} < D_{i,j} \text{ for all } i,j \text{ where } i \neq j, \text{ and } \sum_{i=1}^{n} w_{i,t} = 1\} \tag{10}$$

when there are only proportional transaction costs. For $n = 2$ this reduces to

$$D_- < w_{1,t} < D_+$$

and for $n > 2$ it has the geometry illustrated in Figure 1. We will argue that (10) is a first order approximation to the no-trade region. When considering a rebalance between $w_{i,t}$ and $w_{j,t}$, the other relative weights will be be quite close to the optimal value i.e. $w_{k,t} \approx \tilde{w}_k$ for $k \neq i,j$. Assume the other relative weights satisfy $w_{k,t} = \tilde{w}_k$. Then the border of the no-trade region is a function of $w_{i,t} - w_{j,t}$ only, which again leads to (10).

This approach differs from the dynamic programming approach by Sun *et al.* [12] and Leland [7] who are not able to describe the no-trade region in their papers or Liu [8], who does not have a sufficient number of parameters to describe of the no-trade region flexibly.

The no-trade region R_P, i.e. the optimal values of $D_{i,j}$, is found by optimizing (3). A standard optimization program may be used. In this paper, a very simple and robust optimization is used at the cost of longer computer time. One parameter is changed in each iteration, and only improvements are used further. The integral

i/j	1	2	3	4	5
1	—	0.230	0.091	0.132	0.092
2	0.67	—	0.164	0.052	0.078
3	0.51	0.56	—	0.132	0.219
4	0.51	0.49	0.45	—	0.254
5	0.47	0.52	0.53	0.58	—

TABLE 2. Optimal no-trade region for the example shown in Table 1 with $\gamma = 4$ and proportional transaction costs with $c_{i,P} = 0.001$. Above the diagonal is the length $L = D_{i,j} + D_{j,i}$ of the interval for $w_i - w_j$, and below the diagonal, is the position P of the interval. The no-trade region is then $\tilde{w}_i - \tilde{w}_j - LP < w_i - w_j < \tilde{w}_i - \tilde{w}_j + L(1 - P)$.

strategy	r_U	$E\{W_1\}$	$E\{W_1^{1-\gamma}\}$	C_n	C_c
no rebalance	5.017	1.0654	0.8625	0	0
annual rebalance	5.034	1.0654	0.8625	1	0.00013
monthly rebalance	5.020	1.0649	0.8628	12	0.00041
optimal rebalance	5.042	1.0653	0.8624	30	0.000085

TABLE 3. Comparison of four different rebalancing strategies when there are proportional transaction costs. The parameters are as shown in Table 1 with $\gamma = 4$. The optimal no-trade region is shown in Table 2. r_U is the risk free interest rate corresponding to the utility function. $E\{W_1\}$ and $E\{W_1^{1-\gamma}\}$ are the expected values after 1 year. C_n and C_c denote the annual number of transactions and annual transaction costs, respectively.

i/j	1	2	3	4	5
1	—	0	13%	0.001%	12%
2	0.1%	—	0.1%	3%	7%
3	26%	0.4%	—	0.01%	0
4	0.05%	0.07%	0.05%	—	0
5	26%	13%	0	0	—

TABLE 4. Percentage of the transaction costs from trade between pairs of assets. Above the diagonal is bying asset i and selling asset j.

(3) is evaluated based on 10.000 simulations until 10 years and then the tail is estimated for $t > 10$. This is not large enough to make all the digits in the Tables significant but good enough for any practical purpose. The same realizations of $V_{i,t}$ are used in all iterations of the optimization program in order to make the evaluation of the utility function smooth.

Table 2 shows the no-trade region for $\gamma = 4$. Note how the length L varies between the different combinations of two assets, and the position P varies around the symmetric 0.5 value. Notice that positive correlation increases the size of the no-trade region. This is seen comparing the rebalance of asset 1 and 2 with asset 4 and 5. Figure 1 shows the no-trade region, varying w_1, w_3 and w_5.

Table 3 compares four different strategies and shows that the optimal rebalance strategy gives the highest utility. Highest utility implies that the corresponding risk free interest rate is highest. No rebalance has the highest expected value after one year and almost the same risk aversion weighted expectation as optimal rebalance

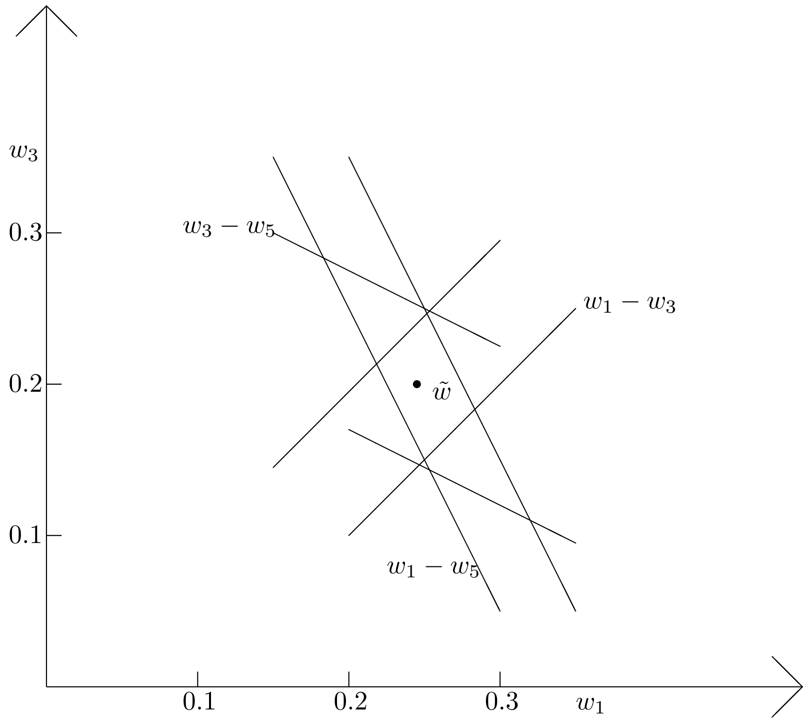

FIGURE 1. No-trade region from Table 2 in the $w_1 \times w_3$ plane, varying w_1, w_3 and w_5 and assuming $w_2 = \tilde{w}_2$ and $w_4 = \tilde{w}_4$. Each of the weight differences $w_1 - w_3$, $w_1 - w_5$ and $w_3 - w_5$ give two parallel lines bounding the no-trade region. The no-trade region is the smallest parallelogram including $\tilde{w}$. Notice that the lines determined by $w_3 - w_5$ do not reduce the no-trade region in this intersection.

after one year. This is due to expected larger ratio of high expectation and high risk assets. Optimal rebalance has reduced transaction costs by 75% compared to monthly rebalance. But the expected annual number it is rebalanced, is almost three times as large as monthly rebalance. In this example annual rebalance is second best followed by monthly rebalance and no rebalance. This order is, of course, case specific. If transaction costs decrease, monthly rebalance improves most and if transaction costs increase, then the portfolio is reduced if a monthly or annual rebalance is used. If the different assets become more positive correlated, rebalance does not help as much making no rebalance a better strategy relative to annual and monthly rebalance. Table 4 shows that for optimal rebalance, the largest transaction costs are due to transactions between asset 1 and assets 3 and 5 and between asset 2 and 5. This is due to differences in expected value and correlation between these assets and the ratios of assets as seen from Table 1. Notice that when $L = D_{i,j} + D_{j,i}$ is large (see Table 2), the probability of a rebalance between i and j is small as is also illustrated in Figure 1. The numbers are based on a simulation and in which some of the combinations never occurred. We may expect all combinations to take place sooner or later, even though a rebalance between asset 3 and 5 will not happen in the plane illustrated in Figure 1. For all pairs, the transaction costs of selling the asset with highest expected value are larger than the cost of buying the same asset. We have also found the no-trade region when the transaction costs increase from $c_{i,1} = 0.001$ to $c_{i,1} = 0.01$. The resulting lengths $D_{i,j} + D_{j,i}$ were approximately doubled.

i/j	1	2	3	4	5
1	—	0.082	0.061	0.039	0.061
2	0.12	—	0.034	0.050	0.047
3	0.049	0.051	—	0.063	0.082
4	0.079	0.032	0.079	—	0.13
5	0.044	0.033	0.12	0.21	—

Table 5. Distance $K_{i,j}$ between the no-trade region with both proportional and flat transaction costs, and only proportional transaction costs.

strategy	r_U	$E\{W_1\}$	$E\{W_1^{1-\gamma}\}$	C_n	C_c
no rebalance	5.017	1.0655	0.8625	0	0
annual rebalance	4.996	1.0655	0.8625	1	0.00062
monthly rebalance	4.444	1.0593	0.8770	12	0.0064
optimal rebalance	5.040	1.0654	0.8624	1.1	0.000063

Table 6. Similar to Table 3 with both proportional and flat transaction costs.

Assume there are both proportional and flat transaction costs. If we focus on only rebalance between two assets at the same time, it is natural to have a no-trade region on the same form as when there are only proportional transaction costs, i.e. (10). Hence, we set

(11)

$$R_F = \{\mathbf{w}_t \mid -E^W_{j,i} < w_{i,t} - w_{j,t} < E^W_{i,j} \text{ for all } i,j \text{ where } i \neq j, \text{ and } \sum_{i=1}^{n} w_{i,t} = 1\}.$$

Following the theory from [6], we should rebalance when outside the no-trade region to an internal surface in the no-trade region. We let the border of R_P defined in (10) be this internal surface since this is where we rebalance when there are no flat transaction costs. Further, let $E^W_{i,j} = D_{i,j} + K_{i,j}/W_t$. This makes the distance between the boundary of the no-trade region and the internal boundary, inverse proportional to W_t. The size of the no-trade region depends on W_t since the value of the portfolio will cover the fee. Note that it is not proved that it is optimal to rebalance to the border of R_P and set the distance $E^W_{i,j} - D_{i,j} = K_{i,j}/W_t$.

If we also consider rebalancing more than two assets at the same time, the no-trade region will only be slightly reduced. However, it will complicate the expressions considerably and increase the number of parameters by a factor of $n-1$. Therefore, we assume rebalancing only two assets at a time.

The optimal values of $K_{i,j}$ are determined similarly as $D_{i,j}$, by optimizing (3). The example in this paper is based on 10.000 simulations of the integral (3) until 10 years and then the tail is estimated for $t > 10$.

Tables 5–6 illustrate the example with both proportional and flat transaction costs. The parameters are as shown in Table 1 with $\gamma = 4$. In addition, the flat transaction cost is $c_{i,F} = 0.0001$ for all i. Note that monthly rebalancing gives very poor results with flat transaction costs. In this case, the optimal strategy rebalance very seldom. The reduction in transaction costs is 99% compared to monthly and 90% compared to annual rebalance.

5. Closing remarks

This paper discusses optimal rebalancing of portfolios with transaction costs and constant risk aversion. We have described techniques for finding the optimal relative weights and the no-trade region. The latter is parameterized by $n(n-1)$ parameters, two for each pair of assets. This parameterization describes of the no-trade region well. If the transaction costs are proportional, with no flat or fixed elements, it is optimal to rebalance to the boundary of the no-trade region whenever the portfolio is outside it. If the transactions costs have flat elements, it is optimal to rebalance to an internal surface in the no-trade region. A full rebalance or a calendar-based rebalance is never optimal.

The theory is illustrated in an example with $n = 5$. Four different strategies, namely, no, annual, monthly, and optimal rebalance were tested using simulations. The transaction costs were reduced by 75% compared to monthly rebalance, which is similar to [6] and better than other papers on optimal rebalance. This depends on the parameters and the utility function. The main contribution to the reduction in transaction costs comes from rebalancing to the boundary of the no-trade region instead of a full rebalance. The exact position of the boundary of the no-trade region is probably not very critical for the transaction costs, but more so for the utility function. The example shows that the size of the no-trade region depends heavily on the properties of the stochastic processes, not only the size of the transaction costs.

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References

[1] C. Atkinson, S. Pliska, and P. Wilmott. Portfolio management with transaction costs. *Proc. R. Soc. London* A **453**, 551–563 (1997).

[2] M.-H. Chang. Hereditary portfolio optimization with taxes and fixed plus proportional transaction costs — Part 1. *J. of Appl. Math. Stoch. Anal.*, Article ID 82753, 33p (2007).

[3] A.H.Y. Chen, and F.C. Jen, The Optimal Portfolio Revisioin Policy, *The Journal of Business*, **44**(1), 51–61, (1971).

[4] P. H. Dybvig. Mean-variance portfolio rebalancing with transaction costs, *Working paper*, http://phildybvig.com/papers/tcost1per2.pdf, Washington University, January 2005.

[5] C. Donohue and K. Yip. Optimal portfolio rebalancing with transaction costs: improving on calendar or volatility-based strategies. *J. Portfolio Management* **29**, 49–63, Summer 2003.

[6] H. Holden and L. Holden. Optimal rebalancing of portfolios with transaction costs. Preprint, Norwegian Computing Center, (2007).

[7] H. E. Leland. Optimal portfolio management with transaction costs and capital gains taxes. *Technical Report*, Haas School of Business, UC Berkeley, (1999).

[8] Hong Liu. Optimal consumption and investment with transaction costs and multiple risky assets. *J. Finance* **59**, 289–338 (2004).

[9] H. Markowitz. Portfolio selection, *J. Finance* **7**, 77–91 (1952).

[10] R. Merton. Optimum consumption and portfolio rules in a continuous-time model, *J. Economic Theory* **3**, 373–413 (1971).

[11] B. Øksendal and A. Sulem. Optimal consumption and portfolio with both fixed and proportional transaction costs. *SIAM J. Control Optim.* **40**, 1765–1790 (2002).

[12] W. Sun, A. Fan, Li-Wei Chen, T. Schouwenaars, and M. Albota. Optimal rebalancing for institutional portfolios, *J. of Portfolio Management*, 33–43, Winter 2006.

[13] M. Taksar, M. Klass, and D. Assaf. A diffusion model for optimal portfolio selection in the presence or brokerage fees. *Math. of Oper. Res.* **13**, 277–294 (1988).

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